## Software tool for cranial orthosis design

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- Need to be designed individually.



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- Contractual research HS7831610 conducted in collaboration with ING corporation spol. s.r.o..


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- Beside that, we also need auxiliary meshes, so-called cages.



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- The model is modified to individual patient based on 3D scan of the head. The scan of head is cropped afterwards by outlines specified by the medical technician.
- The goal of the transformation is a non-rigid deformation of the orthosis body to fit the cropped scan and a rigid transformation of the locking mechanism to its specified position.



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3. MeshDeform modifier replaced by transformation using radial basis function (RBF), which is computationally less expensive.
4. Transformation by RBF also allows easy incorporation of the rigid parts, which MeshDeform modifier cannot implicitly do.

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## Radial basis function

- Function $\varphi: \mathbb{R}^{3} \rightarrow \mathbb{R}^{3}$ that
- exactly interpolates the displacement $u_{j} \in \mathbb{R}^{3}$ of given control points $x_{j} \in \mathbb{R}^{3}, j=1, \ldots, m$,
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- The displacement function is then represented as

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\varphi_{l}(x)=\sum_{i=1}^{n_{p}} \beta_{l, i} p_{i}(x)+\sum_{j=1}^{m} \theta_{l, j} \rho_{x_{j}}(x),
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$$
\left(\begin{array}{cc}
K & B^{\top} \\
B & 0
\end{array}\right)\binom{\theta_{l}}{\beta_{l}}=\binom{u_{l}}{0}
$$

where

$$
\begin{gathered}
K:=\left(\rho\left(\left\|x_{j}-x_{k}\right\|_{\mathbb{R}^{3}}\right)\right)_{j, k=1, \ldots, m} \in \mathbb{R}^{m \times m}, \\
B:=\left(p_{i}\left(x_{k}\right)\right)_{\substack{i=1, \ldots, n_{p} \\
k=1, \ldots m}} \in \mathbb{R}^{n_{p} \times m}
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- We use triharmonic or thin plate spline (TPS)

$$
\rho(r)=r^{3}, \quad \rho(r):=\frac{\Gamma(3 / 2-q)}{2^{2 q} \pi^{3 / 2}(q-1)!} r^{2 q-3}
$$

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- These functions ensure that the non-linear part of the transformation and the linear transformations $L_{q}, q \neq r$ tends to zero as we move towards the $r$-th rigid object.


## Radial basis function with rigid transformation

- The coefficients $\beta_{I}$ for polynomial corrections are replaced by weighted sum of the individual object linear transformations

$$
\mathcal{L}(x)=\sum_{q=1}^{n} w_{q}(x) L_{q}, w_{q}(x)=\frac{v_{q}(x)}{\sum_{r=1}^{n} v_{r}(x)}, \quad v_{q}(x)=\frac{1}{\mathcal{D}_{q}(x)^{\mu}}
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\begin{gathered}
\varphi_{l}(x)=\sum_{i=1}^{n_{p}} \mathcal{L}(x) p_{i}(x)+\sum_{j=1}^{m} \theta_{l, j} \tilde{\rho}_{x_{j}}(x), \\
K \theta_{l}+T=u_{l}, I=1,2,3, \quad T=\left(\begin{array}{c}
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- PCA is statistical method used to estimate the necessary information from the measured data.
- We are able to determine the axes through the use of eigenvalues and eigenvectors of the covariance matrix consisting of a small group of neighboring points.

Rigid transformation


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- OpenMP technology to parallelize transformation.


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- We have also measured computational demands of the algorithm based on the model size.
- For all the tests, configuration of the RBF and the solver was as follows:
- Thin plate spline (TPS) as a kernel function $\rho(r)$.
- Bunch-Kaufman factorization of a symmetric matrix using packed storage has been used as a solver.


## Algorithm speed and possible speed-up by OpenMP on

 multiple CPU cores| CPU cores [-] | 1 | 2 | 4 | 8 | 16 | 24 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Shrink [s] | 0.80 | 0.75 | 0.73 | 0.72 | 0.73 | 0.72 |
| Prepare [s] | 85.36 | 46.44 | 23.32 | 12.30 | 7.96 | 5.63 |
| Solve [s] | 3.94 | 2.65 | 2.18 | 1.96 | 1.88 | 1.88 |
| Transform [s] | 248.07 | 124.19 | 62.29 | 32.56 | 15.75 | 10.87 |
| Total [s] | 338.16 | 174.03 | 88.51 | 47.54 | 26.33 | 19.09 |



## Computation times for different sizes of the model

| Model size [vertices] (1d) | 243897 | 494826 | 1033902 |
| :--- | ---: | ---: | ---: |
| Shrink [s] (2) | 0.24 | 0.72 | 2.61 |
| Prepare [s] (3)-(4) | 1.65 | 4.83 | 18.54 |
| Solve [s] (5) | 0.44 | 1.89 | 10.19 |
| Transform [s] (6) | 2.76 | 8.79 | 38.91 |
| Total [s] (2)-(6) | 5.09 | 16.23 | 70.25 |




## Result model



