Multi-dimensional magneto-hydrodynamic code based on curvilinear finite elements

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Abstract

The magneto-hydrodynamic (MHD) description is used for simulations of all magnetized fluids from the small scale of microfluidics to the largest scales of stars and galaxies. Our main interest lies in the context of the laser plasma research with focus on laboratory astrophysics and inertial fusion. However, the proposed numerical solution is not limited to these areas and can be applied elsewhere. In addition to the basic resistive MHD model, the simulation code applies the two-temperature description, where the electrons have a distinct temperature from the ions. This enables the simulations on the short time scales of the laser–plasma interaction, where the species are not in an equilibrium. Unlike most of the codes in the field, the code PETE2 (Plasma Euler and Transport Equations version 2) relies on high-order curvilinear finite elements, which follow the flow of the matter within the Lagrangian description [1, 2]. An increased numerical efficiency and scalability is achieved this way. Multiple examples of the simulations in different physical scenarios are presented.

Objectives:

- multi-dimensional resistive magneto-hydrodynamics
- Lagrangian formulation on curvilinear finite elements
- high-order finite element discretization
- conservation of energy
- divergence-free magnetic field

Physical model

- resistive magneto-hydrodynamics with $\vec{E} = \eta\vec{j}$
- closure model for the stress tensor $\overline{\overline{\sigma}} = -p\overline{\overline{I}}$
- magnetic stress tensor $\overline{\overline{\sigma}}_B = 1/\mu_0(\vec{B}\vec{B} - \frac{1}{2}\vec{B}^2\overline{\overline{I}})$
- magnetic energy $\epsilon_B = \vec{B}^2/(2\mu_0)$ (in rest frame)

Lagrangian resistive MHD equations [3]:

mass conservation
$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\nabla\cdot\vec{u}$$
momentum conservation
$$\rho\frac{\mathrm{d}\vec{u}}{\mathrm{d}t} = \nabla\cdot(\overline{\overline{\sigma}} + \overline{\overline{\sigma}}_B)$$
Faraday's law
$$\frac{\mathrm{d}\vec{B}}{\mathrm{d}t} = -\nabla\times\vec{E}$$
internal energy conservation
$$\rho\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \overline{\overline{\sigma}} : \nabla\vec{u} + \vec{j}\vec{E}$$
magnetic energy conservation
$$\frac{\mathrm{d}\epsilon_B}{\mathrm{d}t} = \overline{\overline{\sigma}}_B : \nabla\vec{u} - \frac{1}{\mu_0}\vec{B}\cdot\nabla\times\vec{E}$$

Numerical model

functional spaces:

- thermodynamic ($\mathcal{T}$) – L_2-conforming
- kinetic ($\mathcal{K}$) – $(H^1)^d$-conforming (d – dimension)
- magnetic field ($\mathcal{M}$) – H_{div}-conforming / L_2-conforming (2D out-of-plane)
- electric field ($\mathcal{E}$) – H_{curl}-conforming / H^1-conforming (2D out-of-plane)

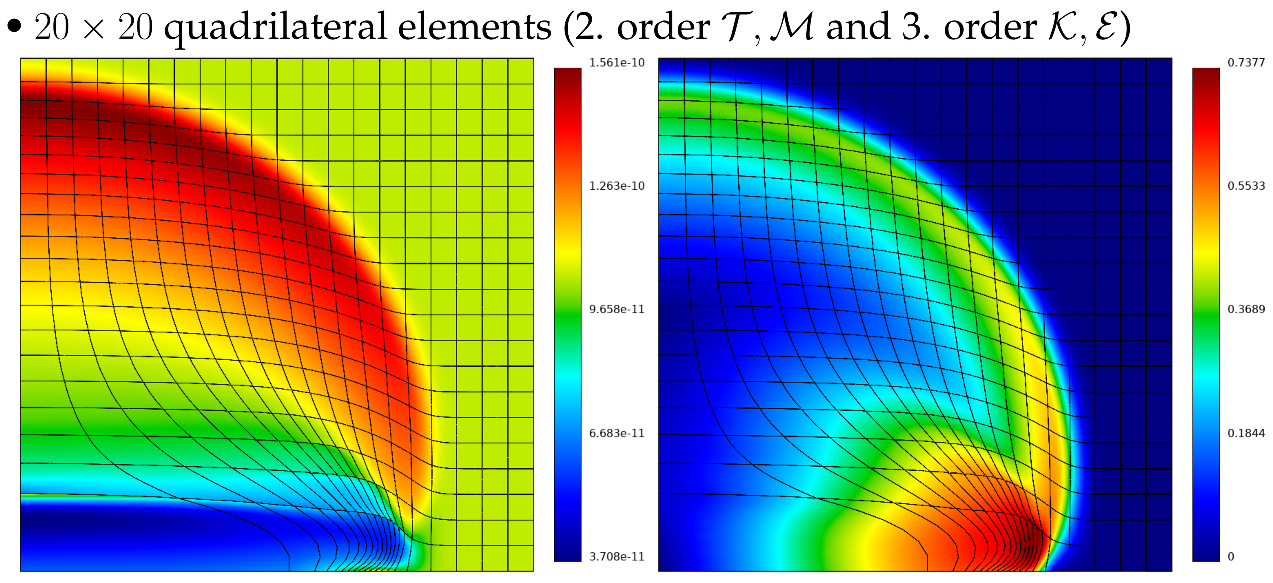
weak formulation:

momentum
$$\int\rho\frac{\mathrm{d}\vec{u}}{\mathrm{d}t}\vec{\psi}\,\mathrm{d}V = \int(\overline{\overline{\sigma}} + \overline{\overline{\sigma}}_B) : \nabla\vec{\psi}\,\mathrm{d}V \qquad \forall\vec{\psi}\in\mathcal{K}$$
mag. field
$$\int\frac{\mathrm{d}\vec{B}}{\mathrm{d}t}\cdot\vec{\Xi}\,\mathrm{d}V = -\int\nabla\times\vec{E}\cdot\vec{\Xi}\,\mathrm{d}V \qquad \forall\vec{\Xi}\in\mathcal{M}$$
el. field
$$\int\frac{1}{\eta}\vec{E}\cdot\vec{\xi}\,\mathrm{d}V = \int\frac{1}{\mu_0}\nabla\times\vec{B}\cdot\vec{\xi}\,\mathrm{d}V \qquad \forall\vec{\xi}\in\mathcal{E}$$
internal en.
$$\int\rho\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\varphi\,\mathrm{d}V = \int\overline{\overline{\sigma}} : \nabla\vec{u}\varphi + \frac{1}{\mu_0}\vec{E}\times\vec{B}\cdot\nabla\varphi + \frac{1}{\mu_0}\vec{B}\cdot\nabla\times\vec{E}\varphi\,\mathrm{d}V \qquad \forall\varphi\in\mathcal{T}$$
mag. en.
$$\int\frac{\mathrm{d}\epsilon_B}{\mathrm{d}t}\varphi\,\mathrm{d}V = \int\overline{\overline{\sigma}}_B : \nabla\vec{u}\varphi - \frac{1}{\mu_0}\vec{B}\cdot\nabla\times\vec{E}\varphi\,\mathrm{d}V \qquad \forall\varphi\in\mathcal{T}$$

Examples

Magneto-hydrodynamic blast (2D)

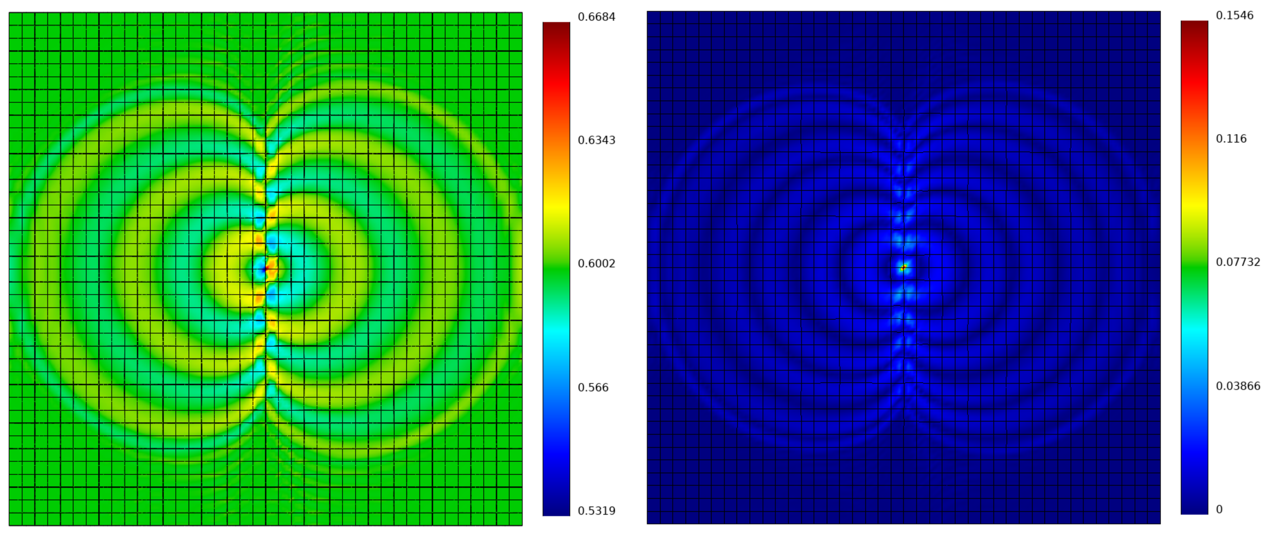
- energy deposited at the origin, initial horizontal magnetic field ($B^2/(2\mu_0) = 2p$)
- low resistivity $\Rightarrow$ field lines freezing in matter
- nearly-spherical magnetosonic wave
- 20×20 quadrilateral elements (2. order $\mathcal{T},\mathcal{M}$ and 3. order $\mathcal{K},\mathcal{E}$)



Magnetic field (arb. units) Velocity (arb. units)

Magneto-hydrodynamic waves (2D)

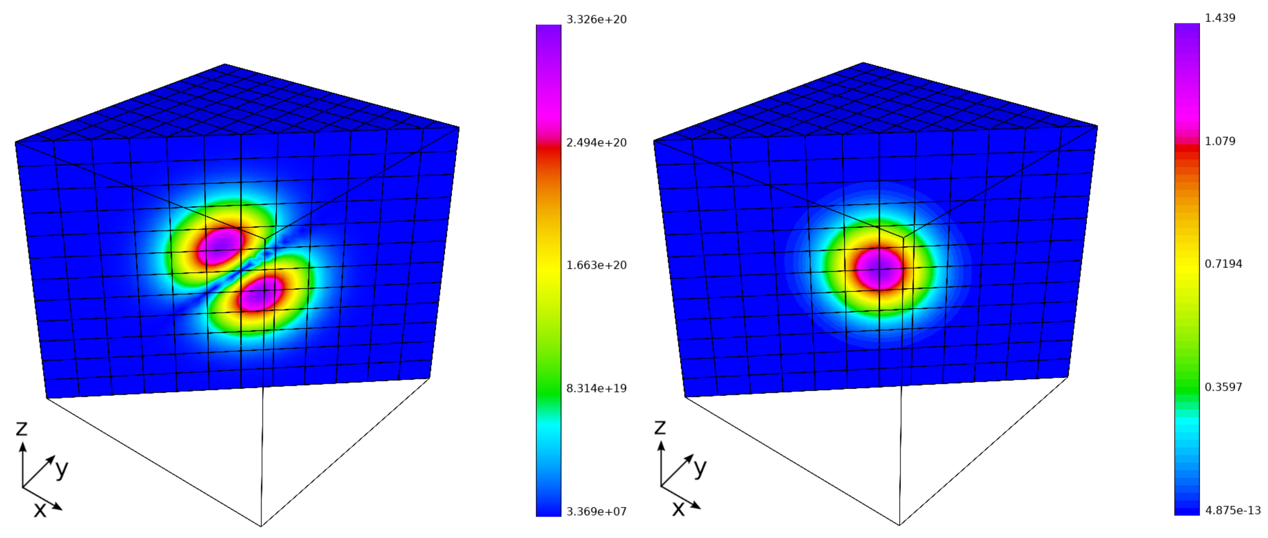
- harmonic oscillation of the velocity field at the center ($v\sim\cos(8\pi t)$)
- vertical magnetic field $\Rightarrow$ vertical Alfven waves, horizontal fast mode
- 40×40 quadrilateral elements (2. order $\mathcal{T},\mathcal{M}$ and 3. order $\mathcal{K},\mathcal{E}$)



Pressure (arb. units) Velocity (arb. units)

Magnetic diffusion (3D)

- initial Gaussian profile of $\vec{B}$ (with orientation $(1,1,1)$)
- diffusion by eddy currents due to finite η
- $12\times 12\times 12$ hexahedral elements (2. order $\mathcal{T},\mathcal{M}$ and 3. order $\mathcal{K},\mathcal{E}$)



Electric field (arb. units) Magnetic field (arb. units)

Conclusions

- multi-dimension curvilinear finite element formulation
- inherently energy conserving + divergence-free magnetic field
- tested on multiple examples

Forthcoming Research

- inclusion of spontaneous magnetic field sources (Biermann battery, thermoelectric effect,...)
- multi-physics simulations with MHD-extended PETE2 code
- parallelization and optimization for different architectures including IT4I infrastructure

References

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