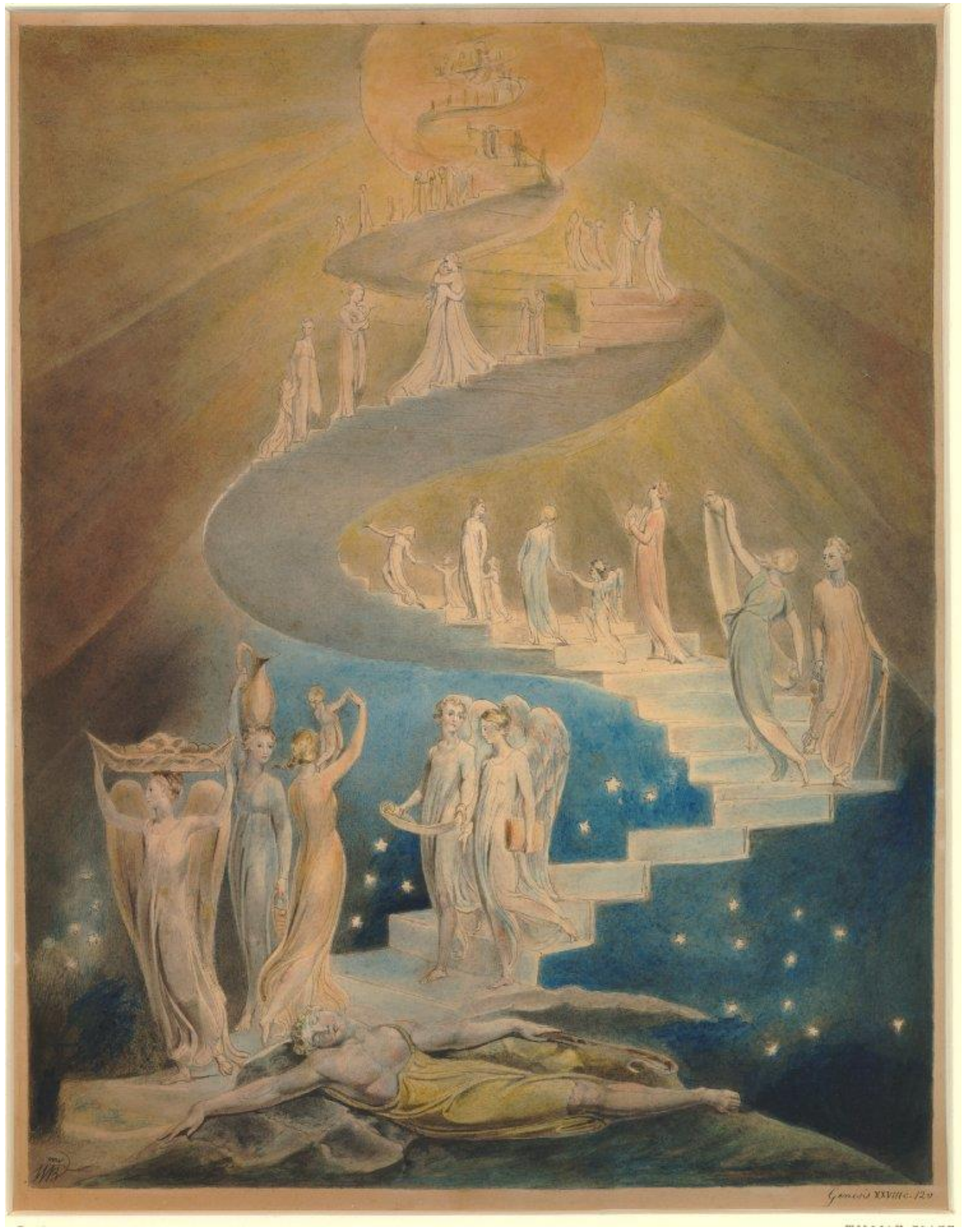
Jacob's Ladder: Prime numbers in 2d

Alberto Fraile, Roberto Martinez, Daniel Fernandez

arXiv.org > math > arXiv:1801.01540



Jacob's Dream by William Blake (c. 1805, British Museum, London)

Prime numbers; open problems

- **Goldbach's Conjecture: Every even $n > 2$ is the sum of two primes.**
- **Twin Prime Conjecture: There are infinitely many twin primes.**
- **Is there always a prime between n^2 and $(n+1)^2$?**
- **Riemann hypothesis**
- **…**

THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

founded in 1964 by N. J. A. Sloane

Search Hints

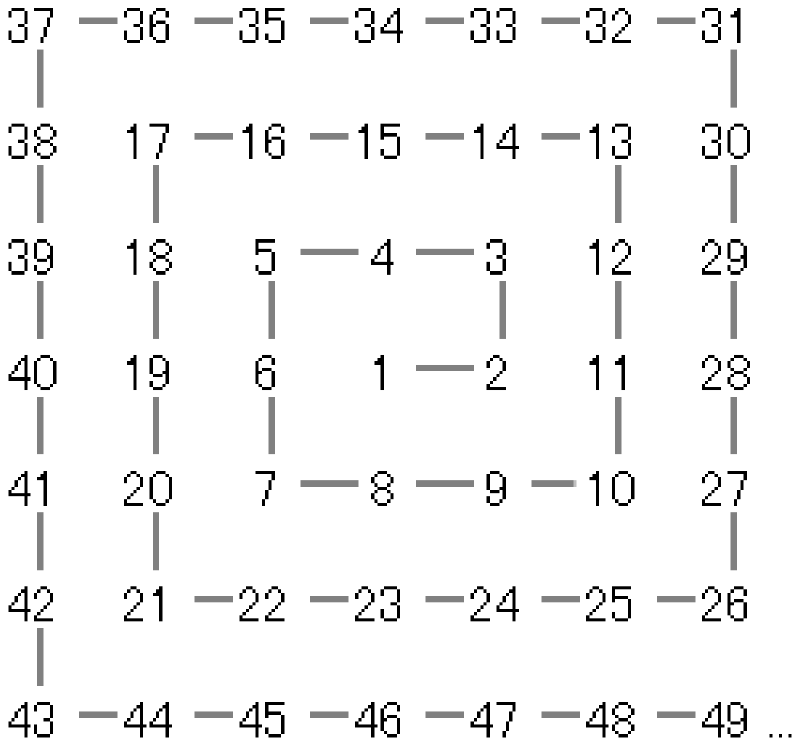
(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A065358 The Jacob's Ladder sequence: a(n) = Sum_{k=1..n} (-1)^pi(k), where pi = A000720. 10

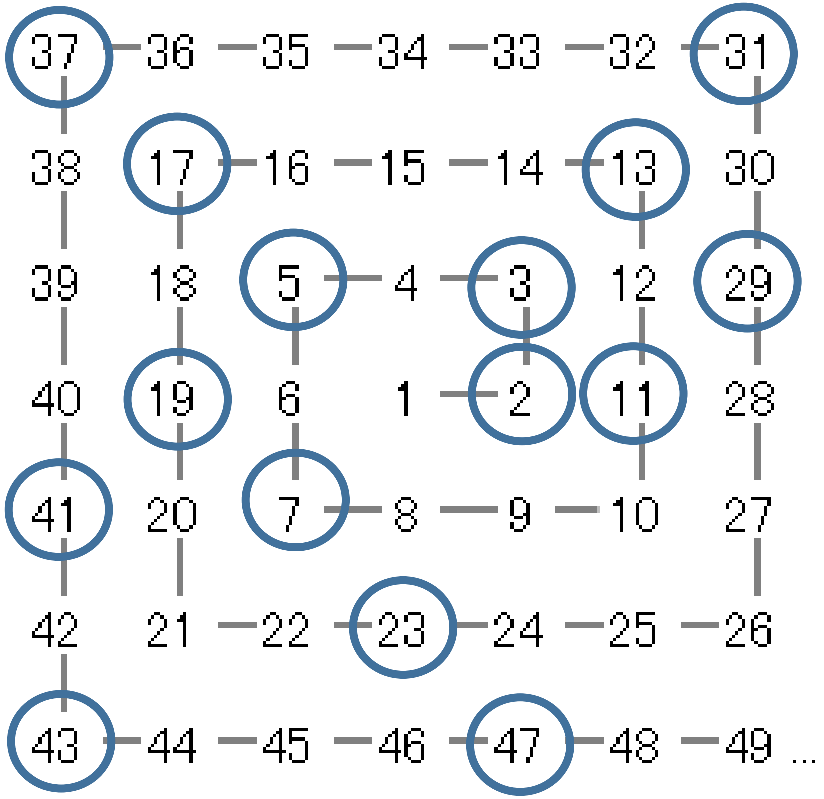
0, 1, 0, 1, 2, 1, 0, 1, 2, 3, 4, 3, 2, 3, 4, 5, 6, 5, 4, 5, 6, 7, 8, 7, 6, 5, 4, 3, 2, 3, 4, 3, 2, 1, 0, -1, -2, -1, 0, 1, 2, 1, 0, 1, 2, 3, 4, 3, 2, 1, 0, -1, -2, -1, 0, 1, 2, 3, 4, 3, 2, 3, 4, 5, 6, 7, 8, 7, 6, 5, 4, 5, 6, 5, 4, 3, 2, 1, 0, 1, 2, 3, 4, 3, 2, 1, 0, -1, -2, -1, 0, 1, 2, 3, 4, 5, 6, 5, 4 (list; graph; refs; listen; history; text; internal format)

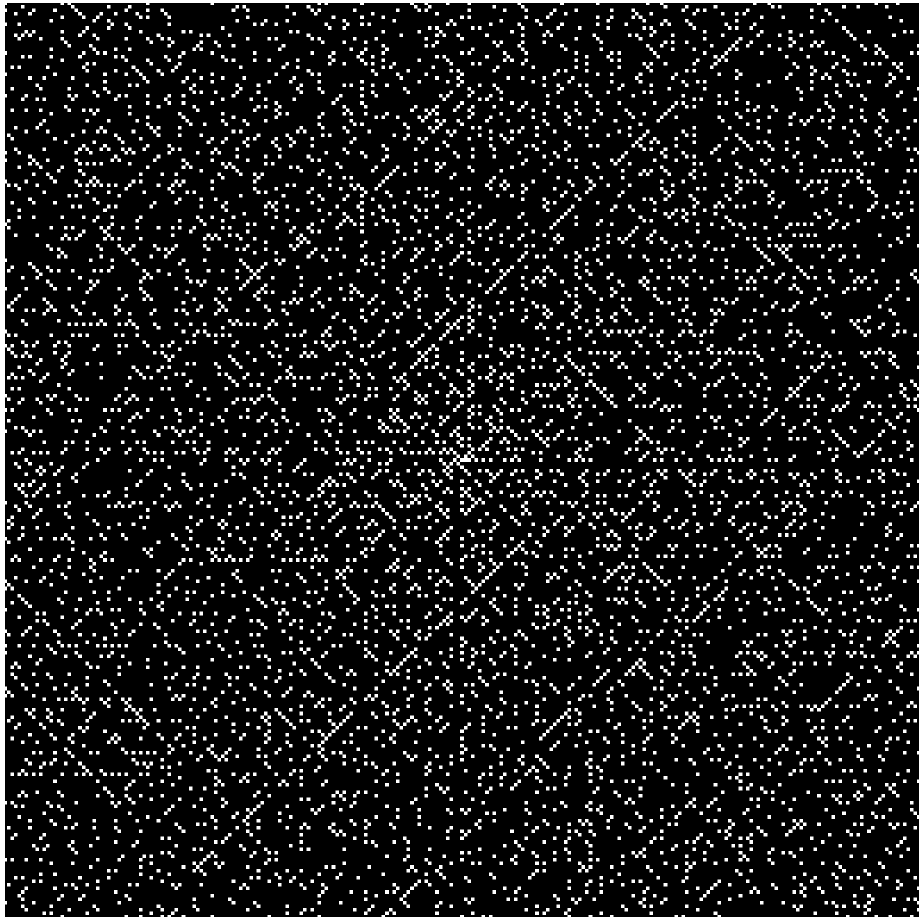
OFFSET	0,5
COMMENTS	Partial sums of A065357.
LINKS	N. J. A. Sloane, Table of n, a(n) for n = 0..10000 (First 1000 terms from Harry J. Smith.)
	Alberto Fraile, Roberto Martínez, and Daniel Fernández, Jacob's Ladder: Prime numbers in 2d, arXiv preprint arXiv:1801.01540 [math.HO], 2017. [They describe essentially this sequence except with offset 1 instead of 0 - N. J. A. Sloane, Feb 20 2018]

Prime numbers in 2d. Ulam spiral



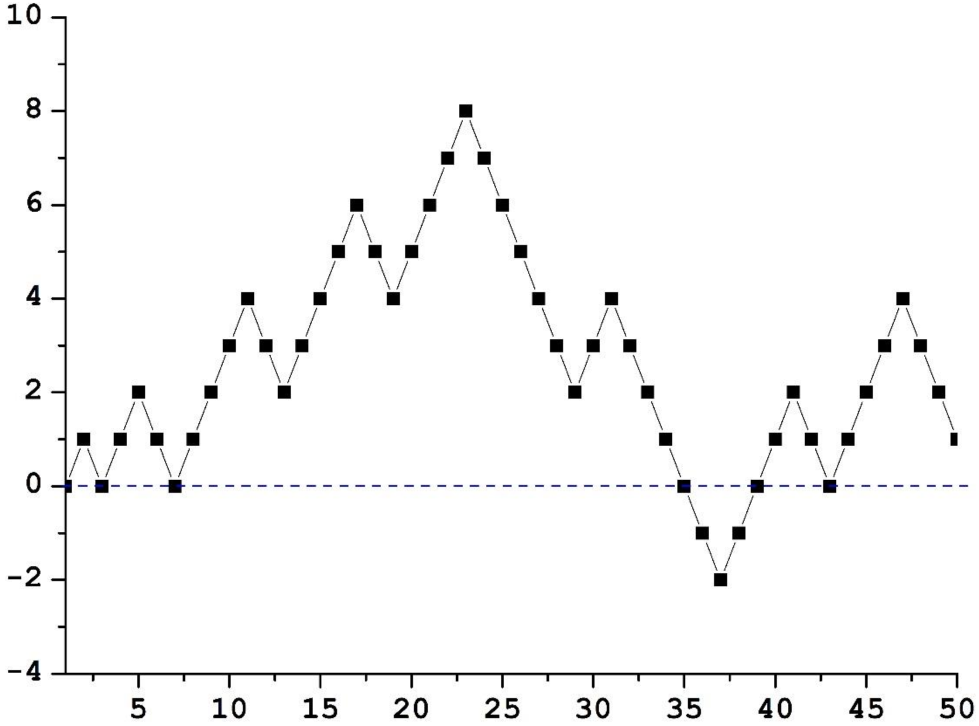
Prime numbers in 2d

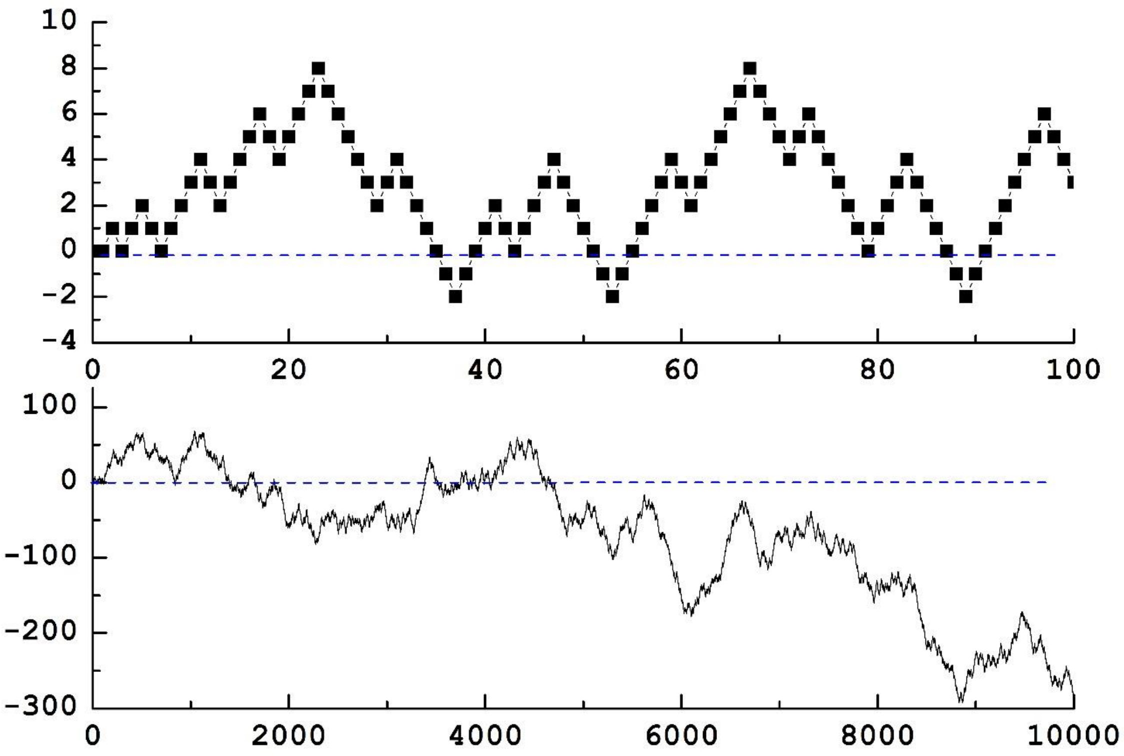


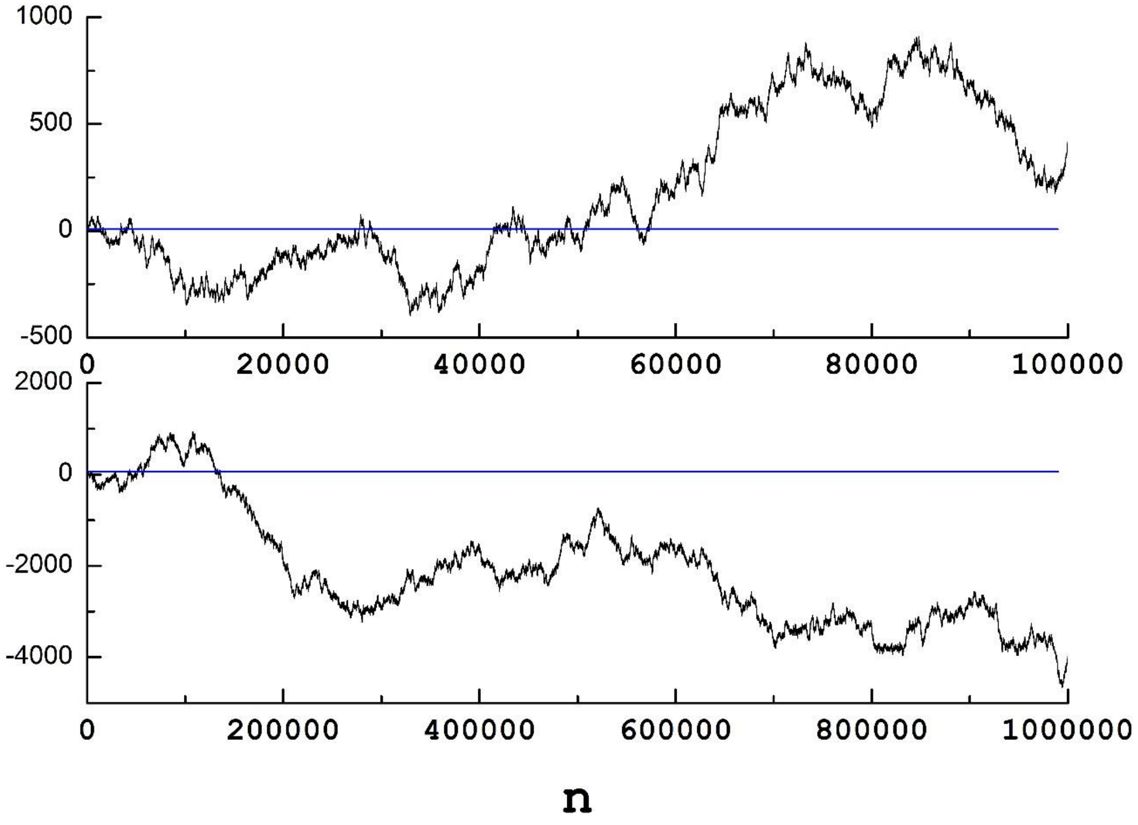


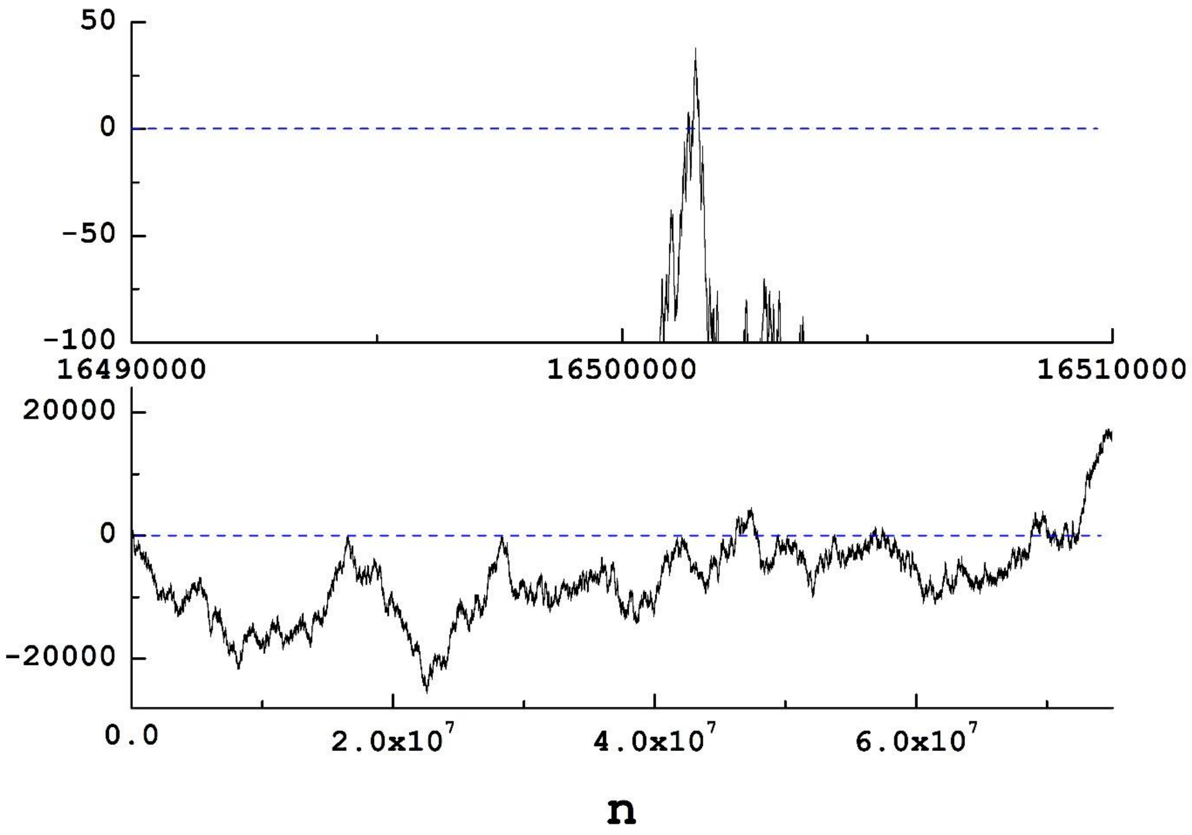
Stein, M. L.; Ulam, S. M.; Wells, M. B. (1964), "A Visual Display of Some Properties of the Distribution of Primes", American Mathematical Monthly, Mathematical Association of America, 71 (5): 516–520

Prime numbers in 2d

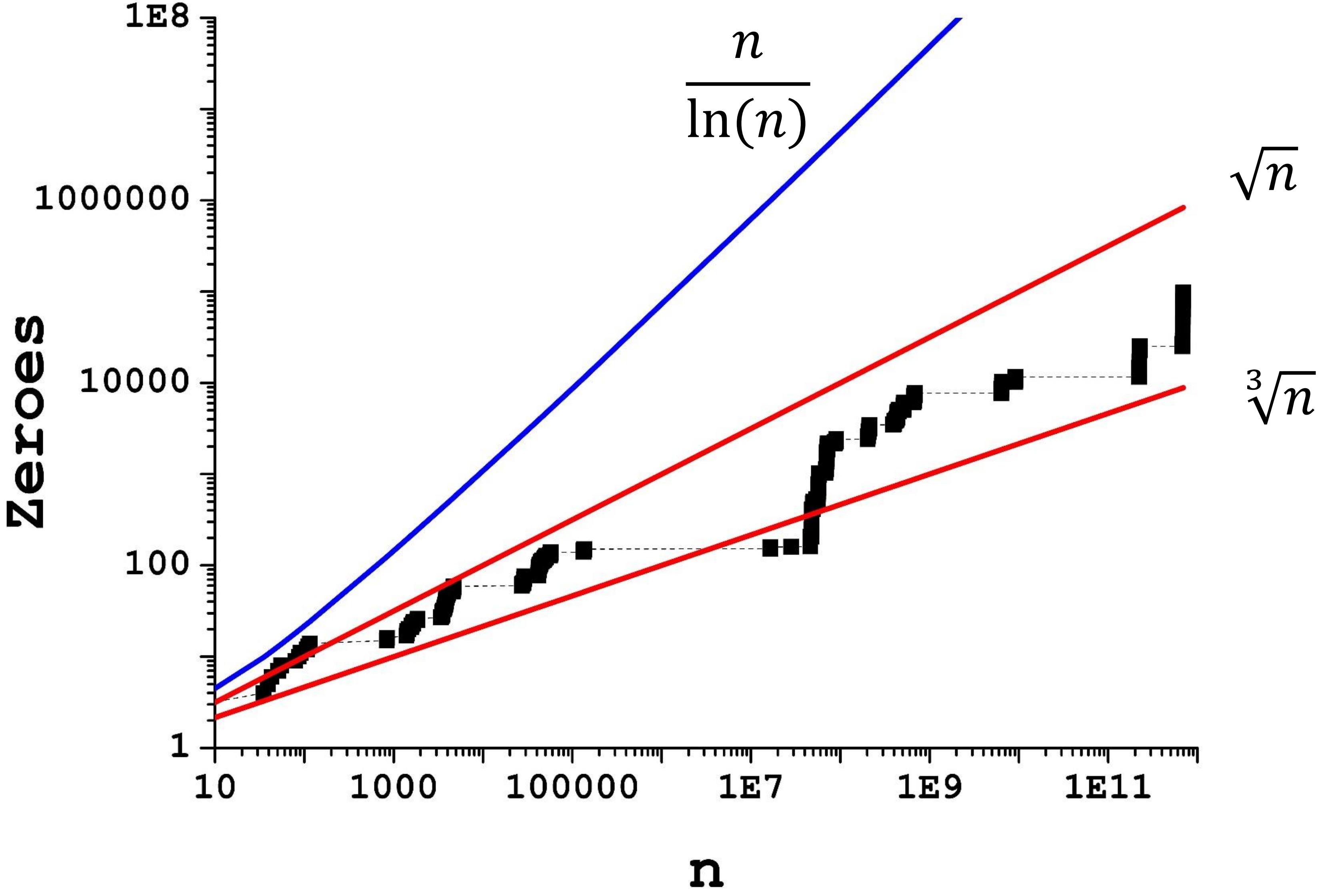



10
8
6
4
2
0
-2
-4
0
20
40
60
80
100
100
0
-100
-200
-300
0
2000
4000
6000
8000
10000


1000
500
0
-500
0
20000
40000
60000
80000
100000
2000
0
-2000
-4000
0
200000
400000
600000
800000
1000000
n


50
0
-50
-100
16490000
16500000
16510000
20000
0
-20000
0.0
2.0x10^7
4.0x10^7
6.0x10^7
n

~200,000 zeroes in $8x10^{12}$



Conjectures

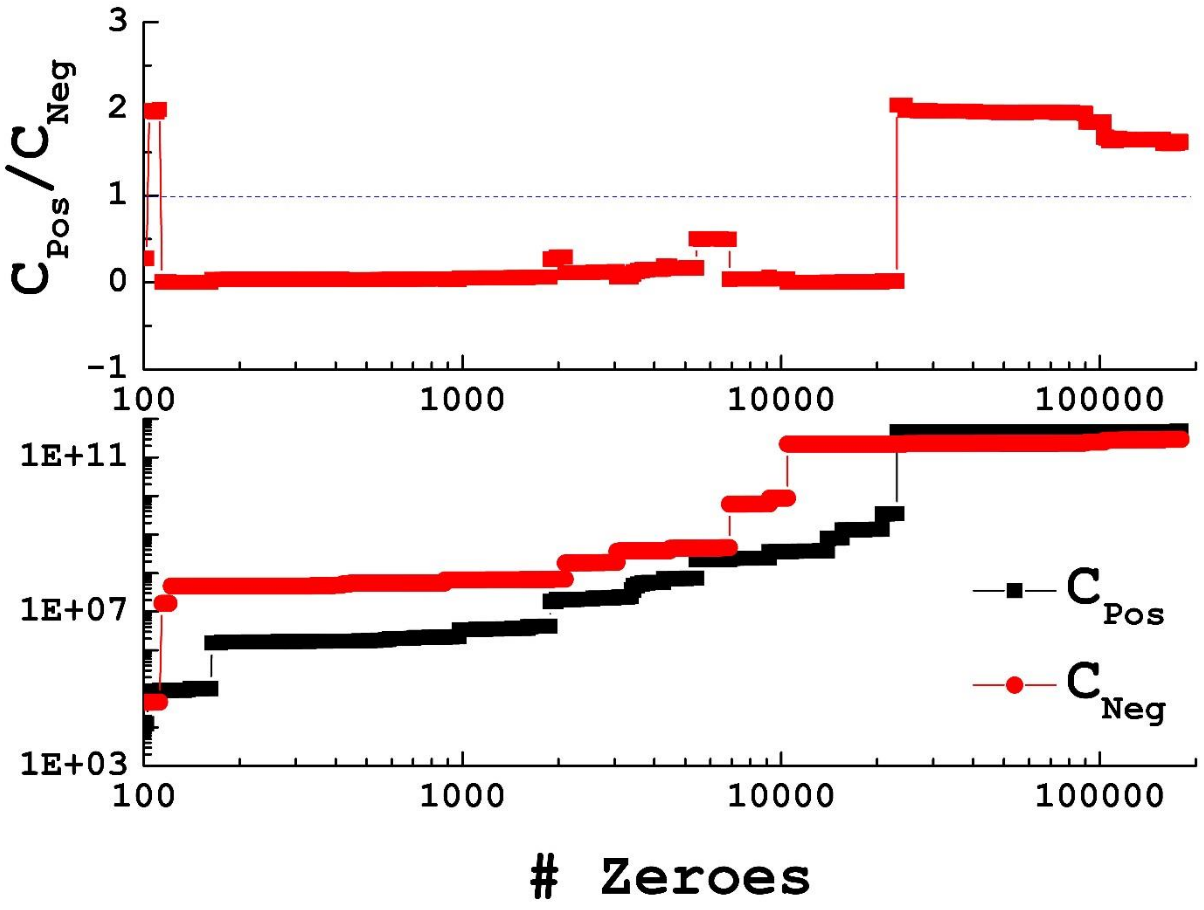
- **I.** The number of cuts (zeroes) in the x axis tends to infinity. i.e, being Z(n) the number of zeroes in the Ladder

$$\lim_{n \to \infty} Z(n) = \infty$$

- **II.** The slope ε(n), of the Ladder is zero in the limit when n goes to infinity.

- **III. A.** the ratio $Area_{up}/Area_{down}$ tends to 1 in the limit $n \to \infty$.
- **III. B.** the ratio between the number of points above and below y = 0 tends to 1 when $n \to \infty$.

Results



C_{Pos}/C_{Neg}
3
2
1
0
-1
100
1000
10000
100000
1E+11
1E+07
1E+03
100
1000
10000
100000
Zeroes
C_{Pos}
C_{Neg}

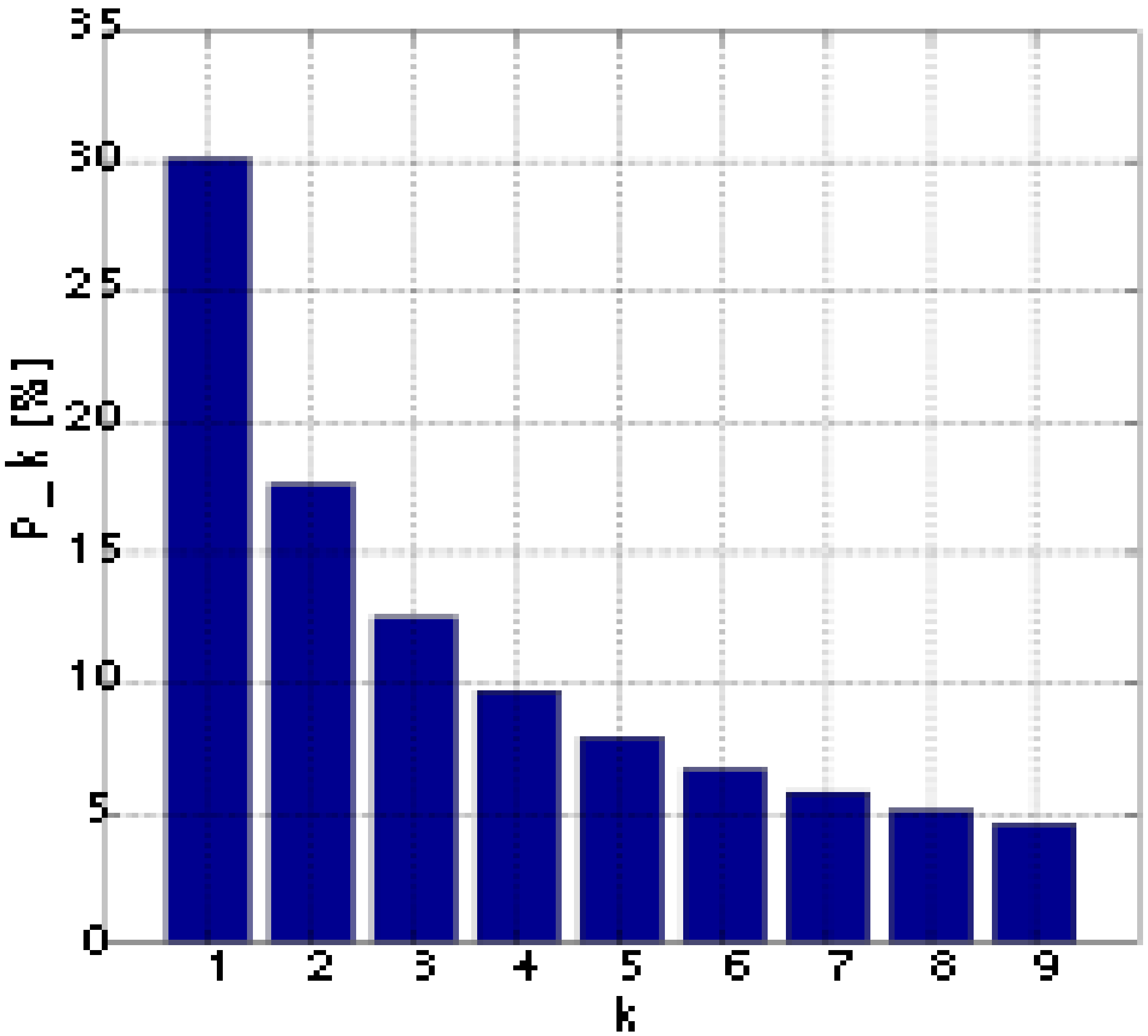
Results I. Benford Law

$$P(d) = \log_{10}(1+1/d)$$

Examples

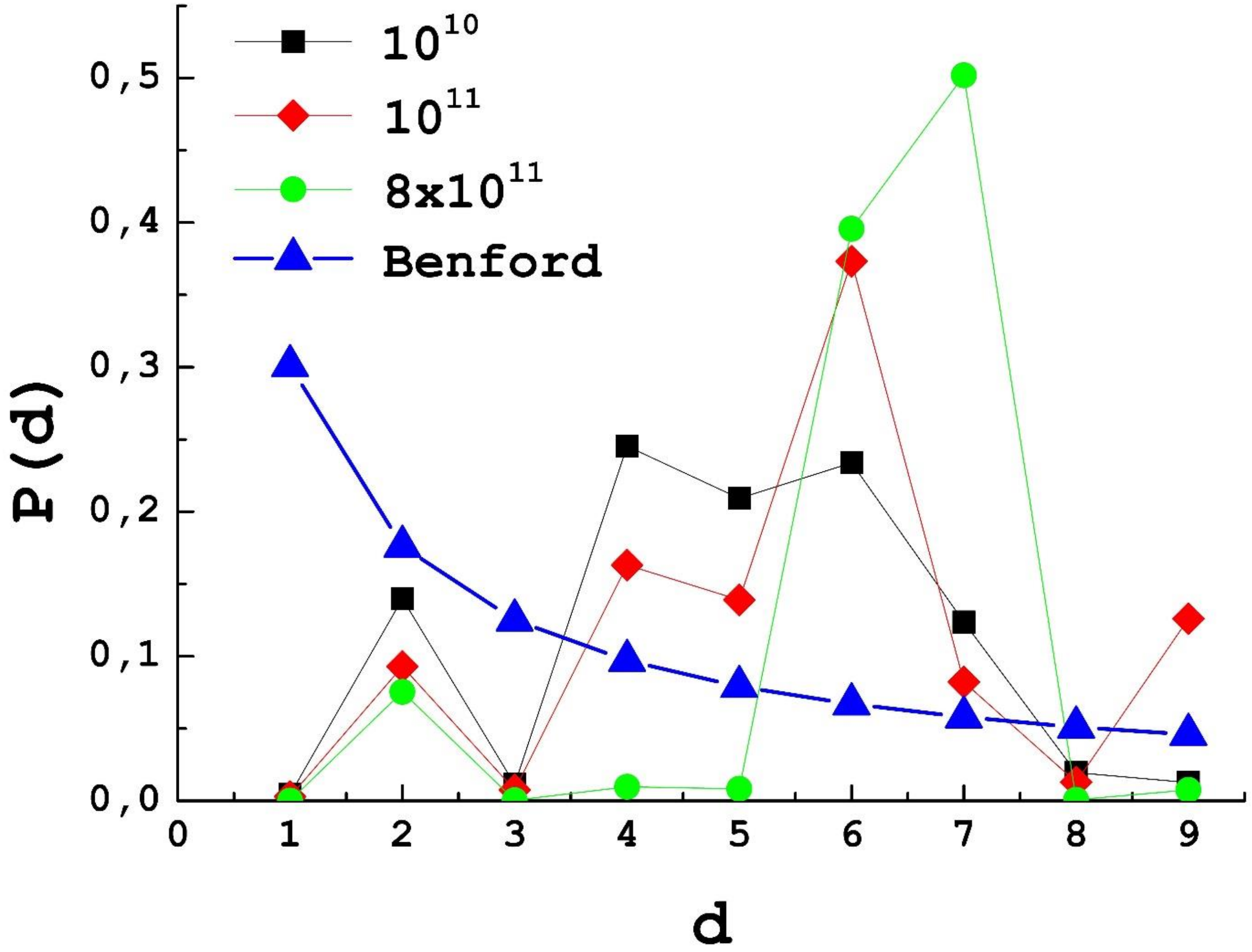
Fibonacci numbers
Factorials n!
Powers n^m
Binomial coefs $\binom{n}{m}$
Etc..

Length of rivers…

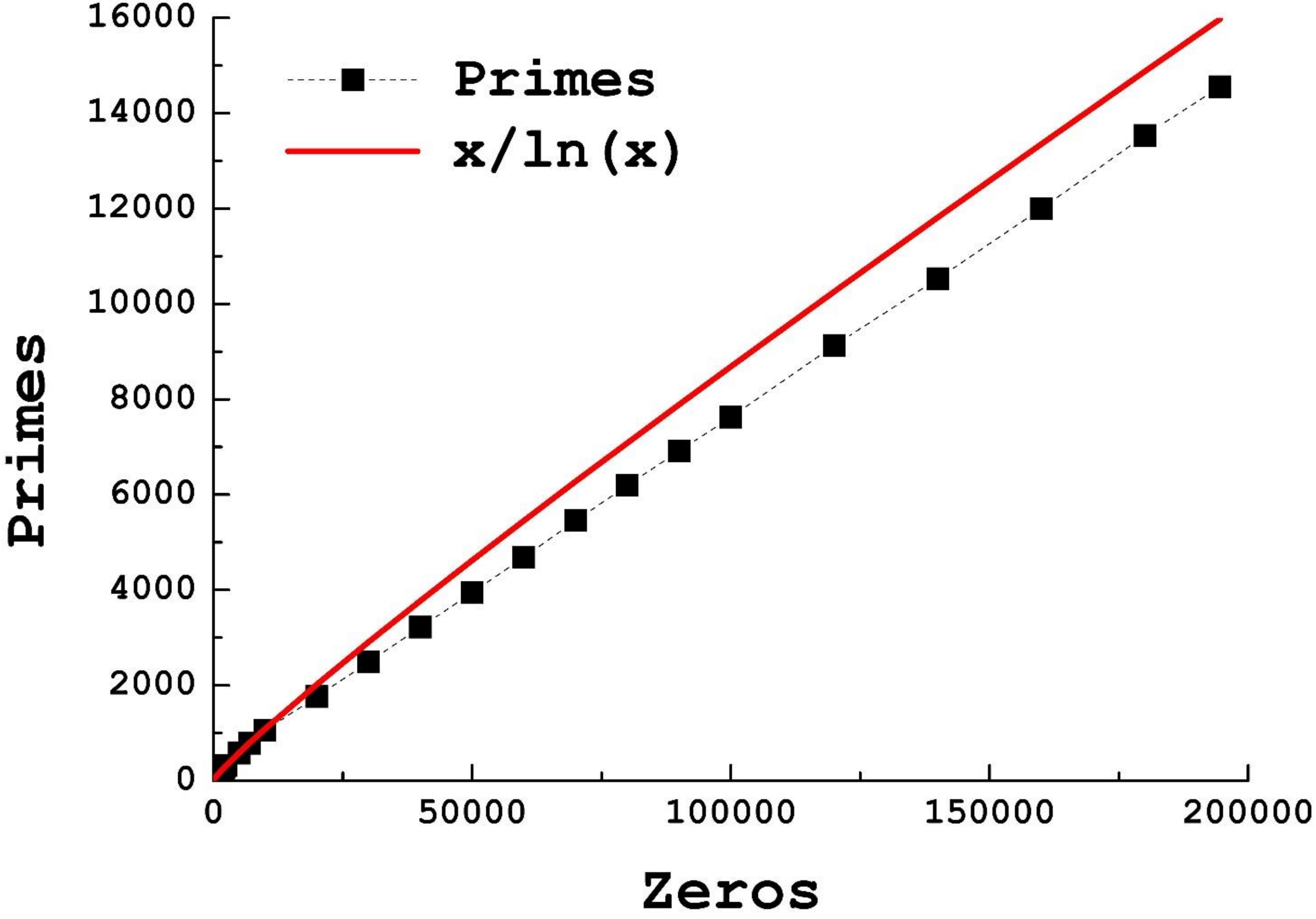


Results I. Benford Law

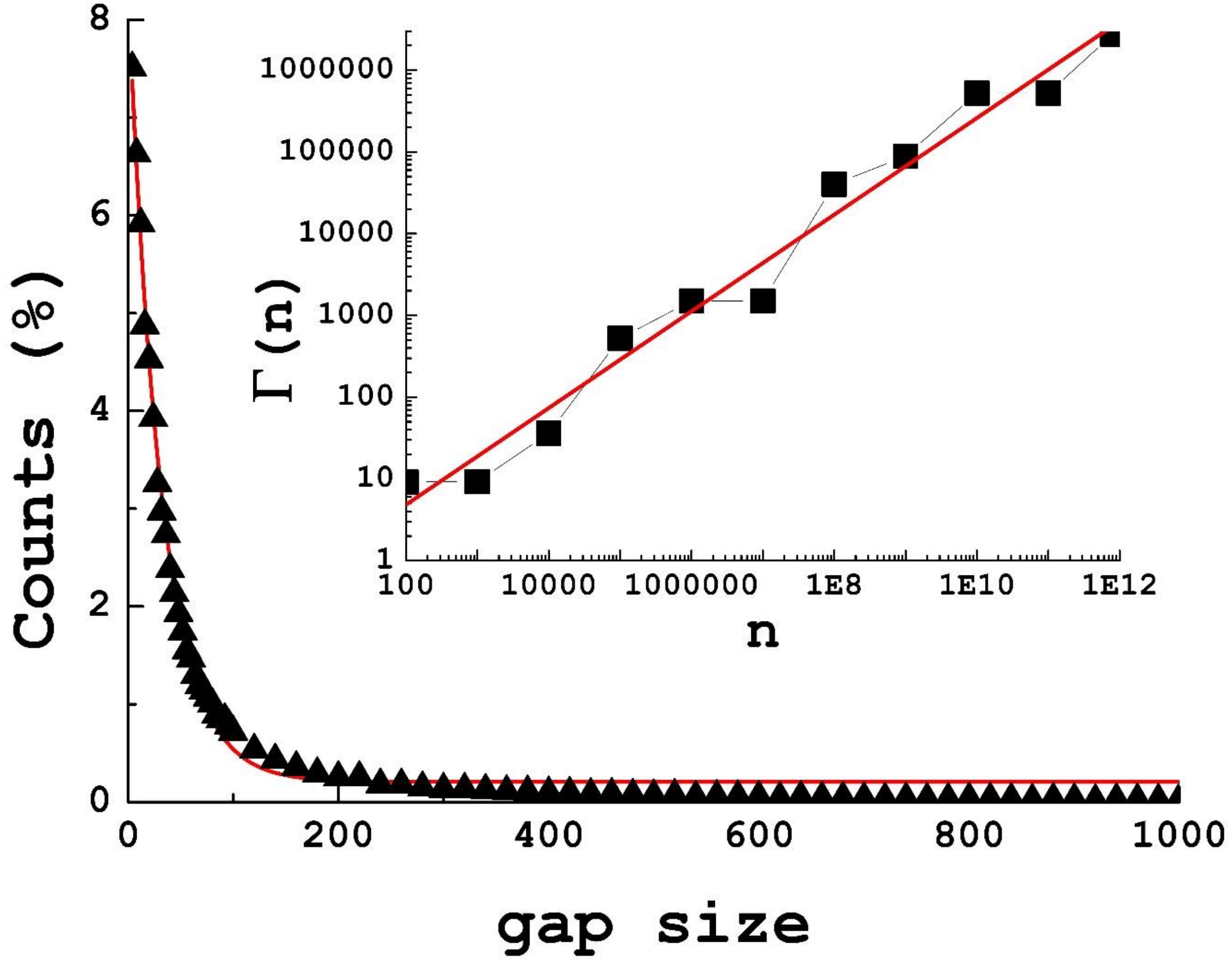
$P(d) = \log_{10}(1+1/d)$



Results II. Prime numbers



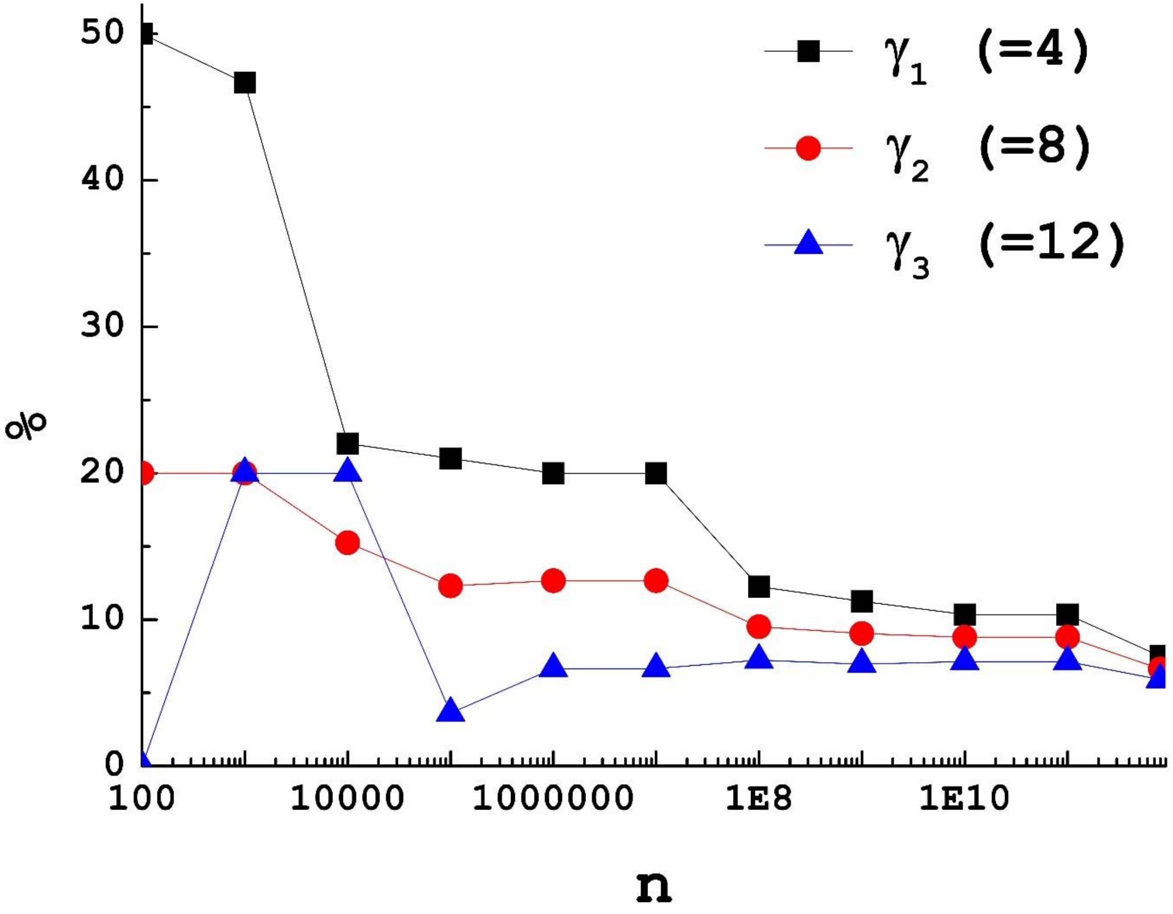
Results III. Gaps



Conclusions

Interval	Zeroes	Primes	$n/\log n$	Diff (%)	Average gap Γ
$[1, 10^2]$	10	5	4.342	13.16	9.2
$[1, 10^3]$	16	6	5.770	3.82	9.25
$[1, 10^4]$	59	21	14.469	31.09	36.20
$[1, 10^5]$	139	36	28.169	21.75	526.57
$[1, 10^6]$	151	37	30.096	18.65	1503.97
$[1, 10^7]$	151	37	30.096	18.65	1503.97
$[1, 10^8]$	2,415	313	310.034	0.947	40170.11
$[1, 10^9]$	7,730	846	863.41	-2.058	887722.55
$[1, 10^{10}]$	11,631	1,161	1,242.438	-7.014	523588.07
$[1, 10^{11}]$	11,631	1,161	1,242.438	-7.014	523588.07
$[1, 8 \cdot 10^{11}]$	194,530	14,556	15,973	-9.734	2750072.04

Conjecture IV. Gamma1 =4 for all n?



Thank you for your attention

Conjecture II