# Inversion of the satellite-observed tidal magnetic fields in terms of 3-D upper-mantle electrical conductivity

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## Summary

The interaction of the oceanic tidal flows with the Earth's main magnetic field provides a powerful natural source of electromagnetic energy suitable for sub-oceanic upper-mantle electrical conductivity sounding. We have developed a new frequency-domain, spherical harmonic-finite element approach to the inverse problem of global electromagnetic (EM) induction. It is set up for an effective inversion of satellite-observed tidally-induced magnetic field in terms of three-dimensional structure of the electrical conductivity in the sub-oceanic upper mantle. The numerical code is parallelized using the OpenMP standard and it uses either the Math Kernel Library (MKL) or AMD Optimizing CPU Libraries (AOCL) to compute the Fourier transform effectively.

In order to demonstrate that the new approach can successfully reconstruct the 3-D upper mantle conductivity, we performed synthetic tests using a 3-D conductivity model WINTERC-E (Fullea et al., 2021) as a testbed. The WINTERC-E model is independent of any EM data and thus it represents an ideal target for synthetic tests of the 3-D EM inversion. In the next step, we proceed to the inversion of satellite-derived (Swarm) models of tidal magnetic signatures. We explore different datasets and different regularization settings in the inversion.

## 3-D inversion setup

### Forward solver

- EM induction equation in the frequency domain
- zero external-field boundary condition at the surface $r = a$
- spherical-harmonic parameterization in angular coordinates
- 1-D finite elements in radius
- weak formulation with divergence-free constraint by Lagrange multipliers
- matrix-free iterative BiCGStab(2) solver with 1-D preconditioner

### Inverse solver

- 13 layers in the upper mantle
- $4^\circ \times 4^\circ$ to $10^\circ \times 10^\circ$ blocks depending on data resolution
- only oceanic grid cells are considered
- $10^4$–$10^5$ parameters (log-conductivities) in model vector $\boldsymbol{m}$
- LMQN minimization of $\chi^2(\boldsymbol{m}) + \lambda R^2(\boldsymbol{m})$ with adjoint sensitivities
- optimal regularization parameter $\lambda$ found by L-curve inspection

## Conclusions

- general methodology is sound
- truncation degree of modern data products is sufficient
- data noise (3% on all degrees) can corrupt reconstructed conductivity
- discrepancy between different models of tidal signatures is of the same order as the effect of 3-D conductivity variations

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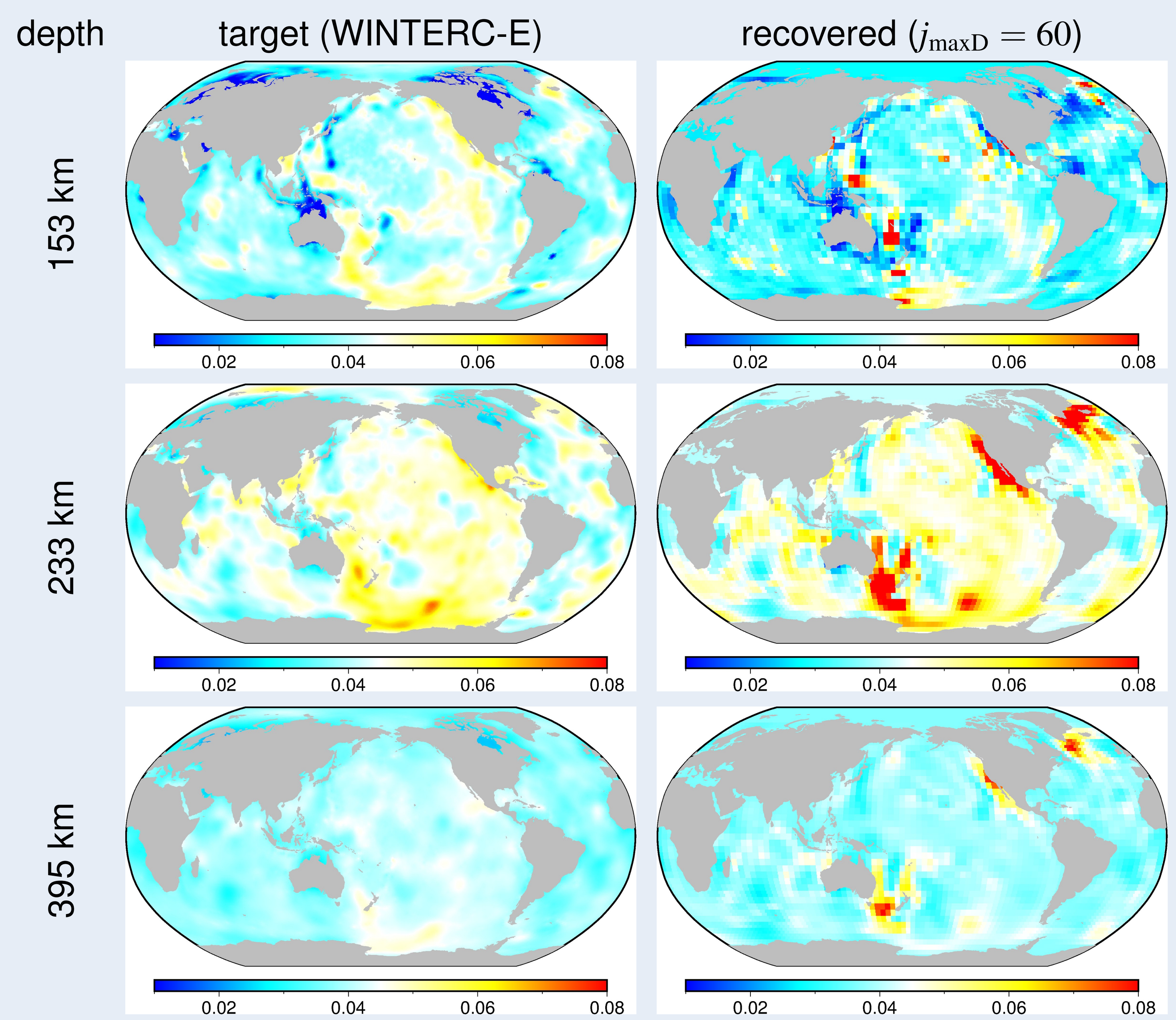
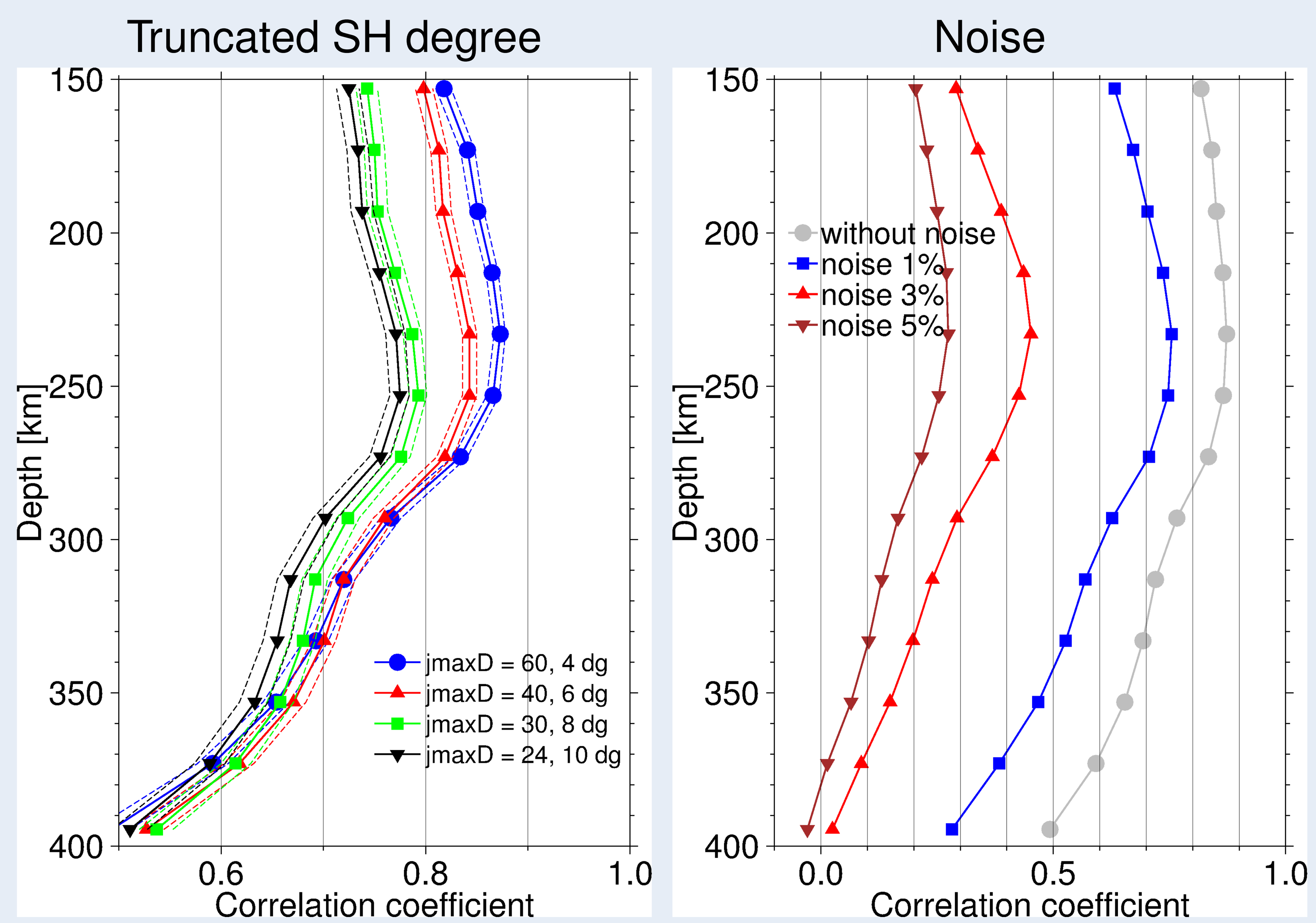
## Synthetic tests

### Target model recovery

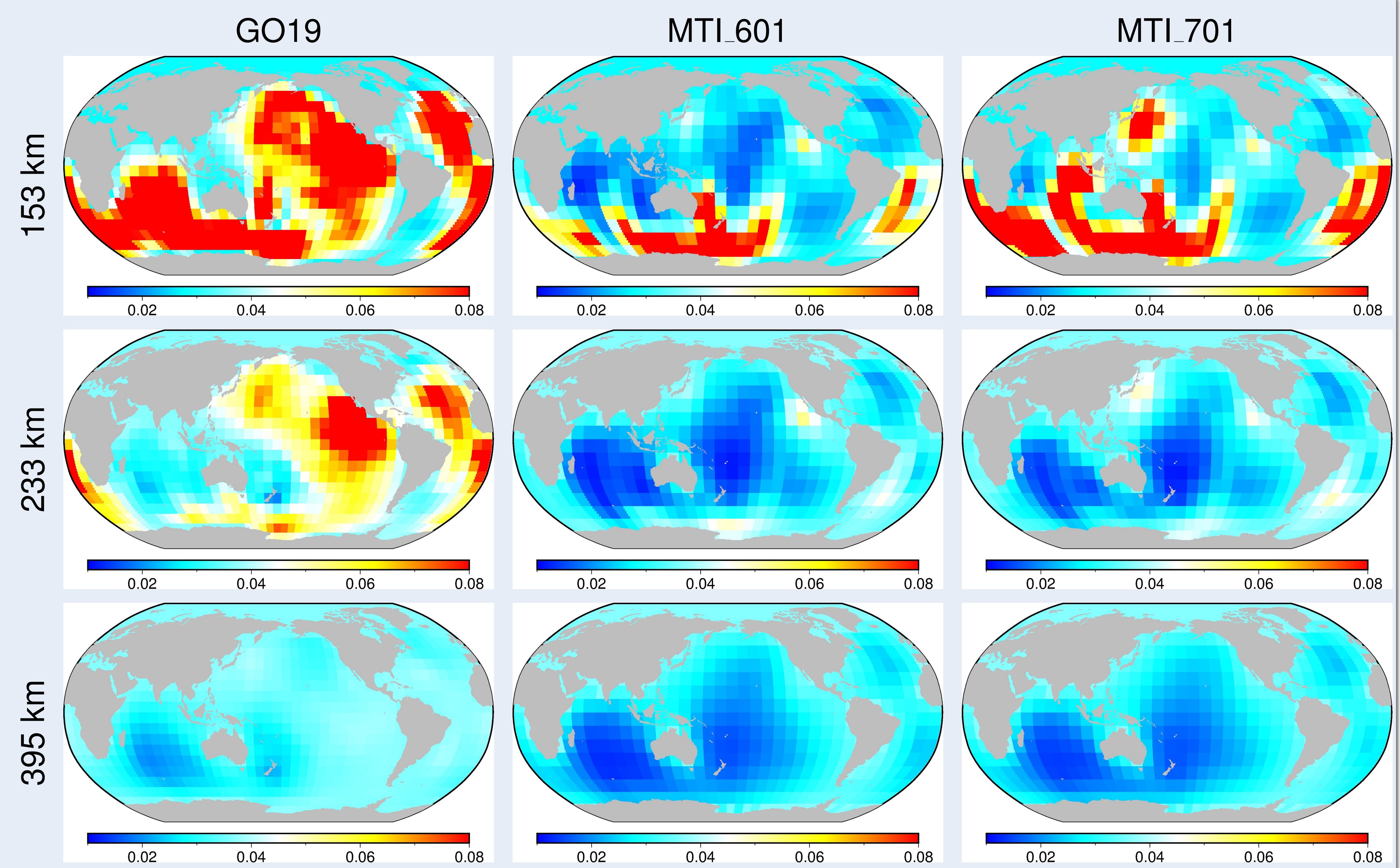



### Impact of data quality

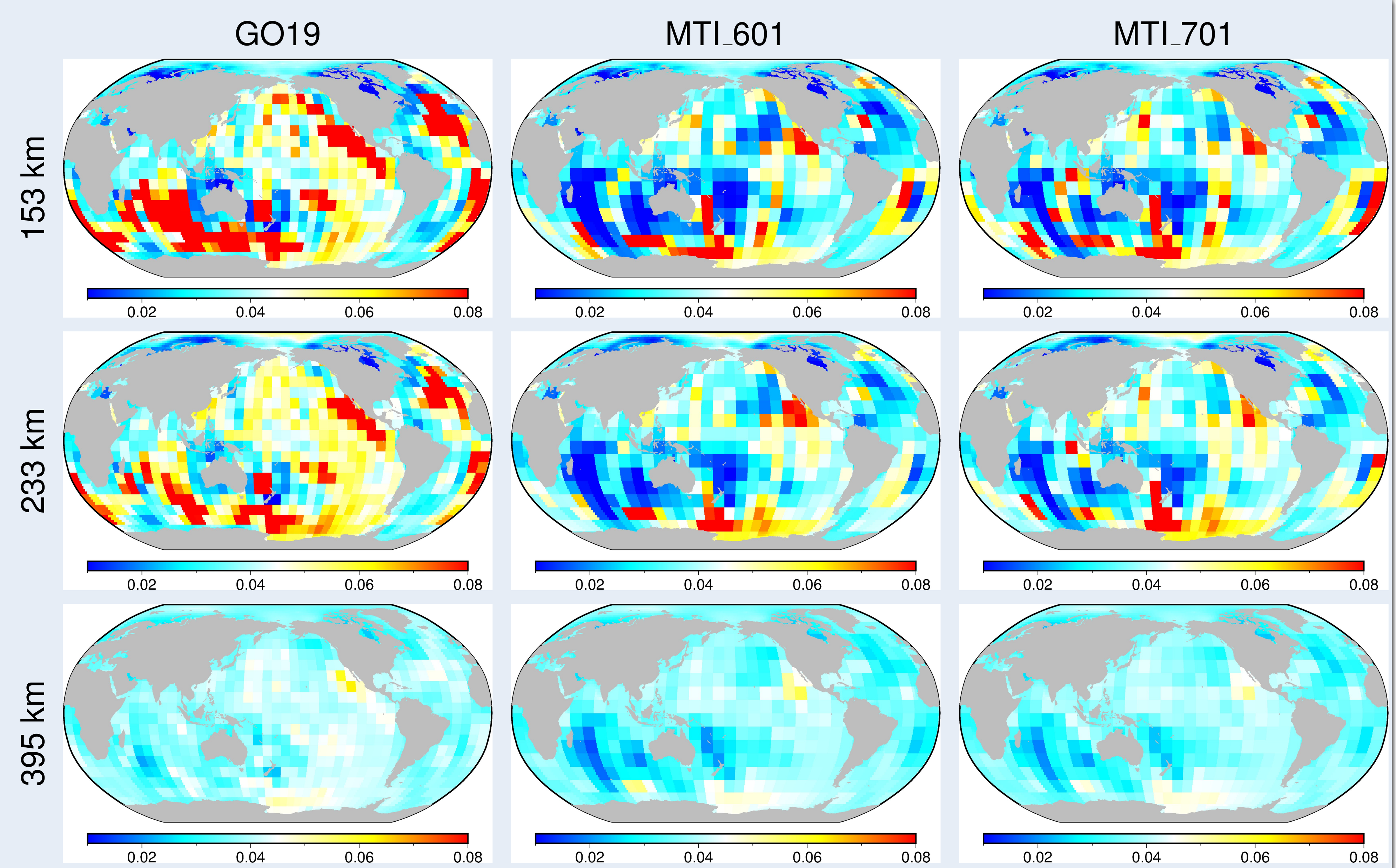



## Inversion of Swarm-based datasets

### Horizontally smoothed models




### WINTERC-E regularized models

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