Preventing Deadlock in Non-uniform-timestep Distributed Pseudospectral Method

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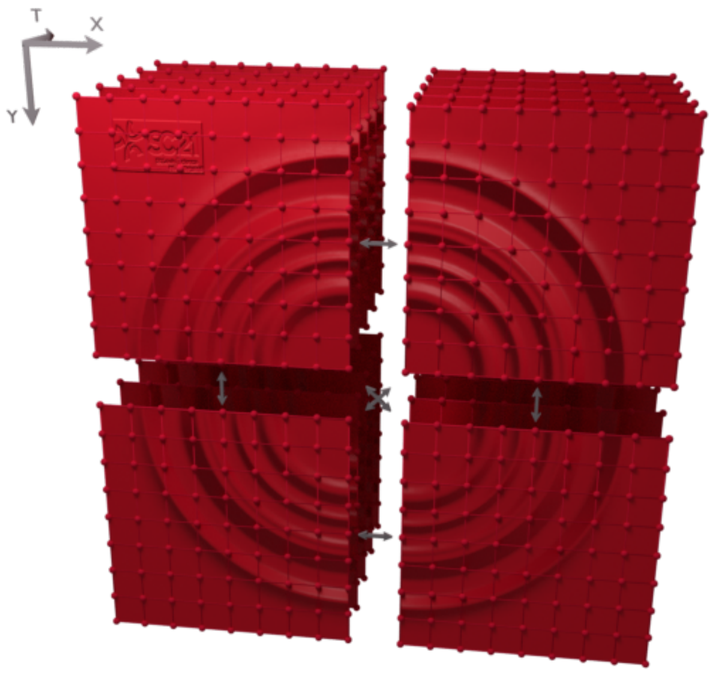
Overview

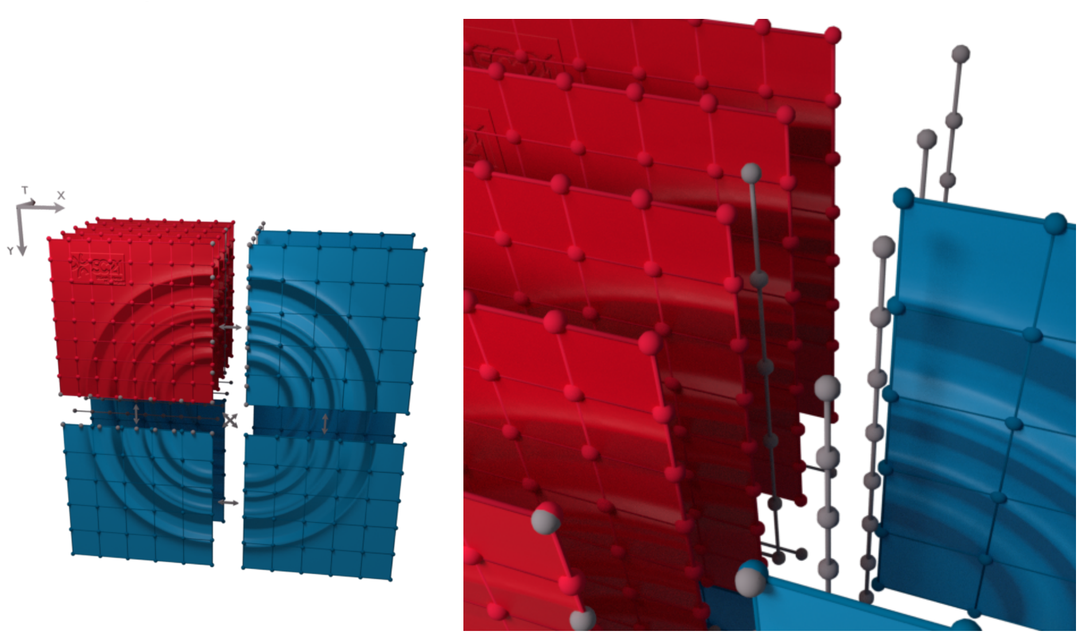
The Local Fourier Basis (LFB) method reduces the amount of global Fast Fourier Transform (FFT) imposed communication by evaluating gradients only locally within the subdomain. Subdomains partially overlap to facilitate conveying of information.



In addition to communication reduction, the LFB method enables some additional modifications and improvements which were not possible in the traditional Fourier pseudospectral method.

Multiple Time Resolutions

In the LFB method, the temporal resolution no longer has to be constant. By having finer timestep only where required, it is possible to save computation time.



To provide the required information for subdomains with different timestep, interpolation in time have to take place. This is done by modification of the integration scheme. In our experimental implementation using a leapfrog method, interpolation is done by simply altering timestep size. We call this method a Non-uniform-timestep Distributed Pseudospectral Method.

Numerical Model

Applied to a system of coupled PDEs describing simplified wave propagation, the modified LFB method uses local FFT to calculate required derivatives in combination with the leapfrog integration scheme to march forward in time.

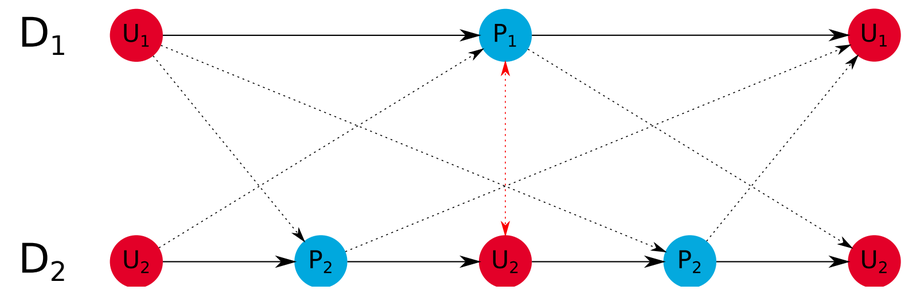
$$U_{step} : u^{n+\frac{1}{2}} = u^{n-\frac{1}{2}} - A\Delta t \frac{1}{\rho}\mathbb{F}^{-1}\left(ike^{-ik\Delta x/2}\mathbb{F}\left(p^n\right)\right)$$

$$P_{step} : p^{n+1} = p^n - A\Delta t \rho c^2 \mathbb{F}^{-1}\left(ike^{ik\Delta x/2}\mathbb{F}\left(u^{n+\frac{1}{2}}\right)\right)$$

It can be seen that P_{step} depends on data from previous U_{step} and vice versa. Operator A denotes a modification of timestep size due to temporal resolution difference in the overlap region.

Deadlock Problem and Solution

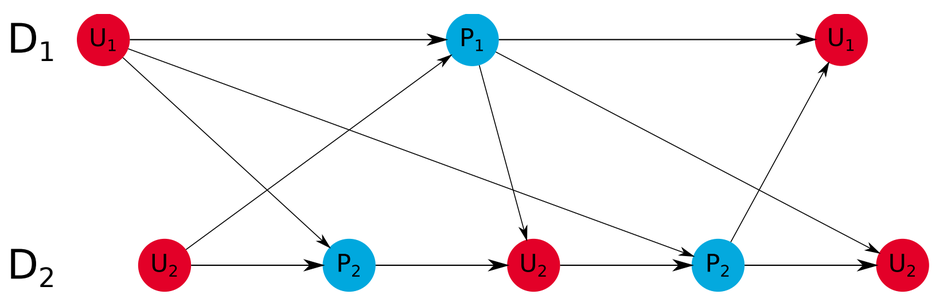
Each sub-step requires data from the neighboring subdomain (depicted with dotted lines) during overlap exchange. To minimize the introduced error, the tendency is to compute values as late as possible. Under specific conditions, this leads to a deadlock, represented by the cycle in the graph.



A simple and effective solution is to define global ordering relation based on timestamps of sub-steps and arbitrary chosen relation $P^n < U^n$. This is equivalent to the definition of generating sub-steps as follows:

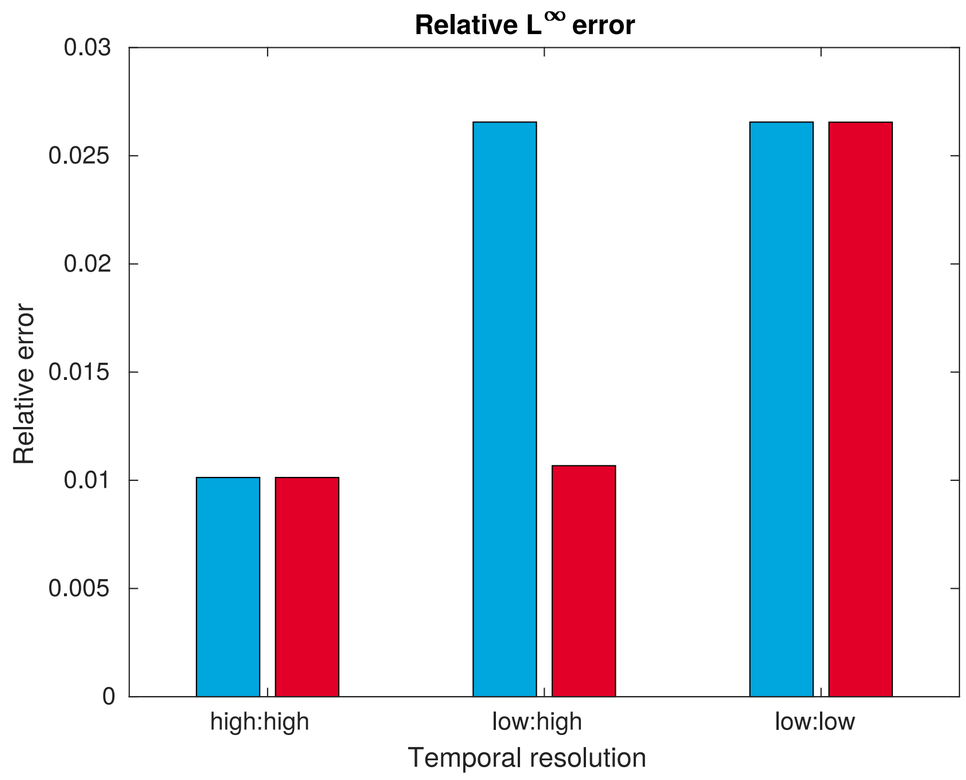
$$G(U_a^n) = \max\{m | P^m \in D_b \wedge m \leq n\}$$

$$G(P_a^n) = \max\{m | U^m \in D_b \wedge m < n\}$$



Accuracy Evaluation

Temporal interpolation was tested on a simple example using two subdomains simulating a homogeneous linear medium. In this case, the interpolation has a negligible effect on the overall accuracy of the simulation.



Conclusion

In this poster, we demonstrated the core principle of the Non-uniform-timestep Distributed Pseudospectral Method which is based on interpolation in time. This is done by additional integration in the overlap region with a modified scheme and/or timestep.

We also demonstrated that under some circumstances there is a possibility of deadlock. The deadlock can arise whenever there is a cycle in the dependency graph. A simple solution to eliminate this possibility is to define a global ordering relation in between sub-steps.

Preliminary tests suggest that a new modification of the LFB method is capable of increasing the efficiency of large-scale distributed simulation by optimizing temporal resolution without significantly decreasing the overall simulation accuracy.