EURO^2

Basic Quantum Computing Algorithms and Their Implementation in Qiskit

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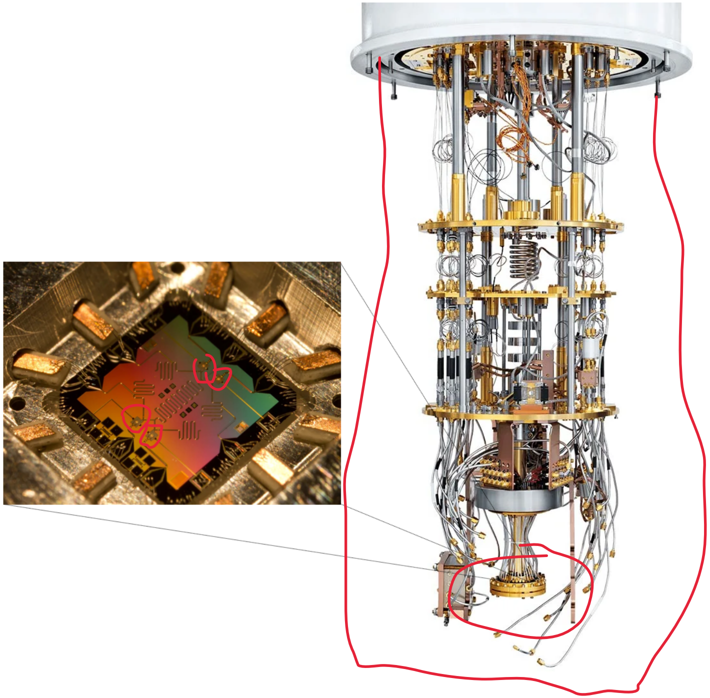
Part I

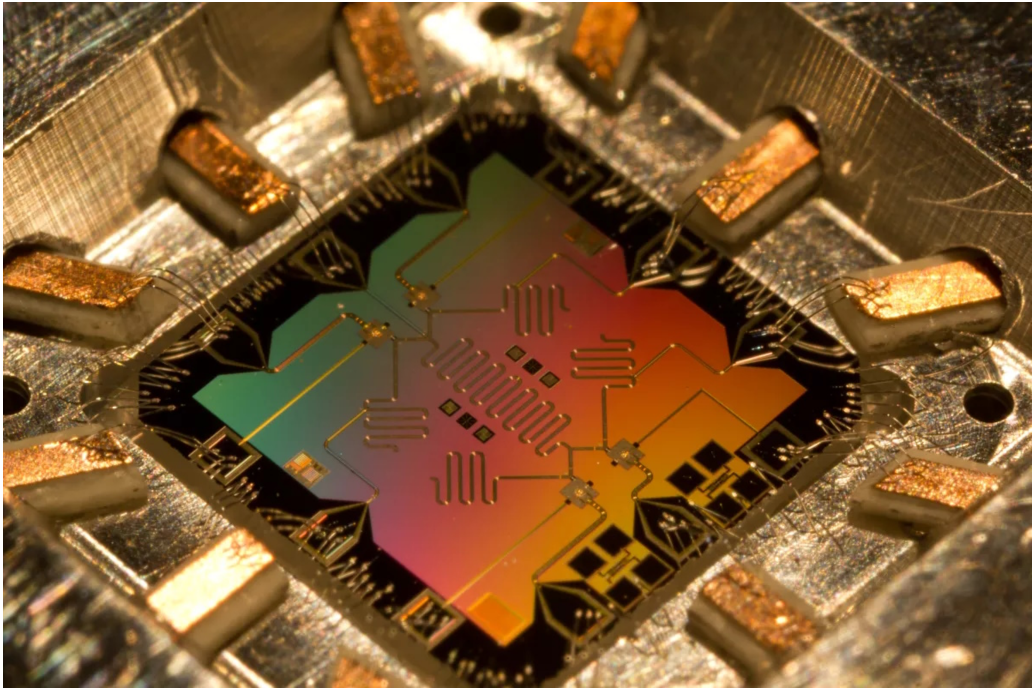
Introduction to Quantum Computing

Hardware

Superconducting technology:

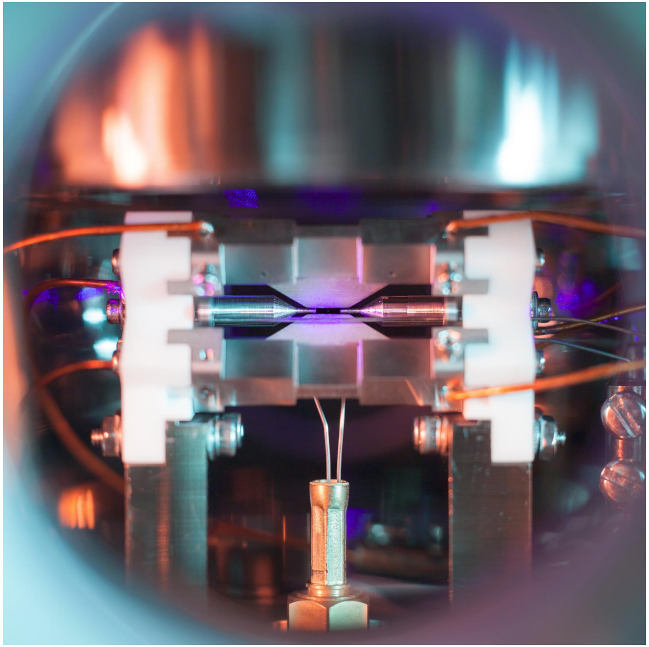


Hardware

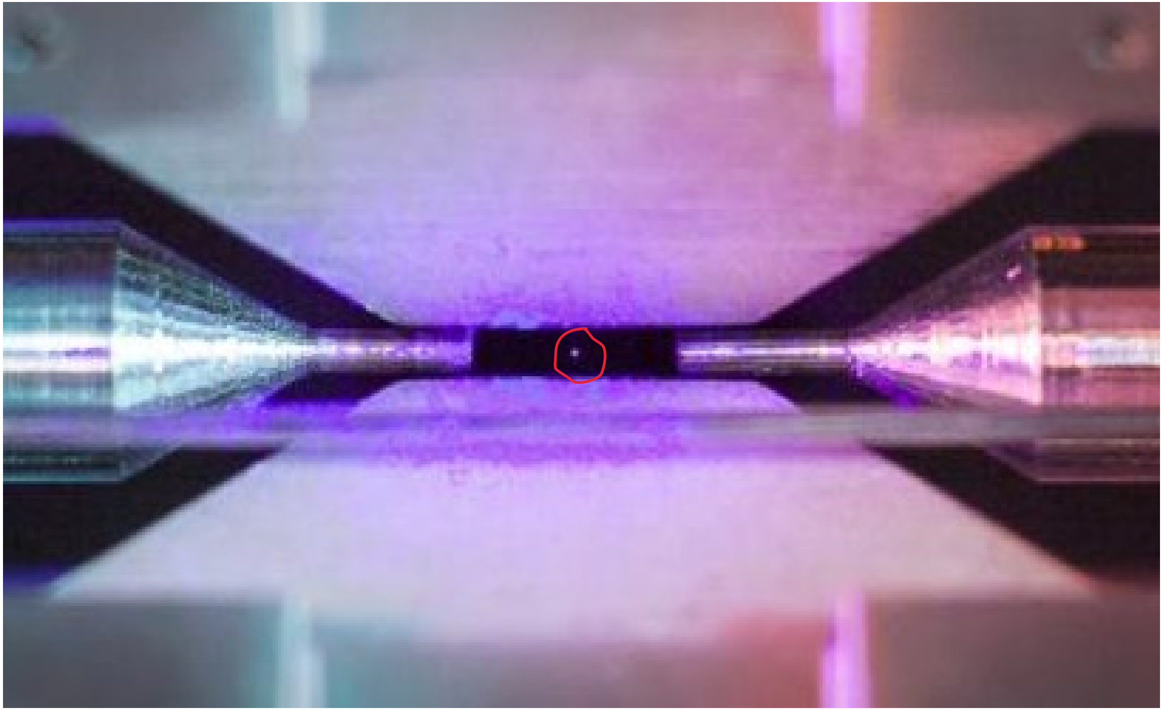


Hardware

Trapped-ion technology:



Hardware



Qubit

ϕ ... PHASE

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

$$\alpha = \cos\frac{\theta}{2}$$

$$\beta = e^{i\phi}\sin\frac{\theta}{2} = (\cos\phi + i\sin\phi)\sin\frac{\theta}{2}$$

$$\Pr(|0\rangle) = |\alpha|^2 = \cos^2\frac{\theta}{2}$$

$$\Pr(|1\rangle) = |\beta|^2 = |e^{i\phi}|^2\sin^2\frac{\theta}{2} = \sin^2\frac{\theta}{2}$$

$$\Pr(|0\rangle) + \Pr(|1\rangle) = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} = 1$$

Z

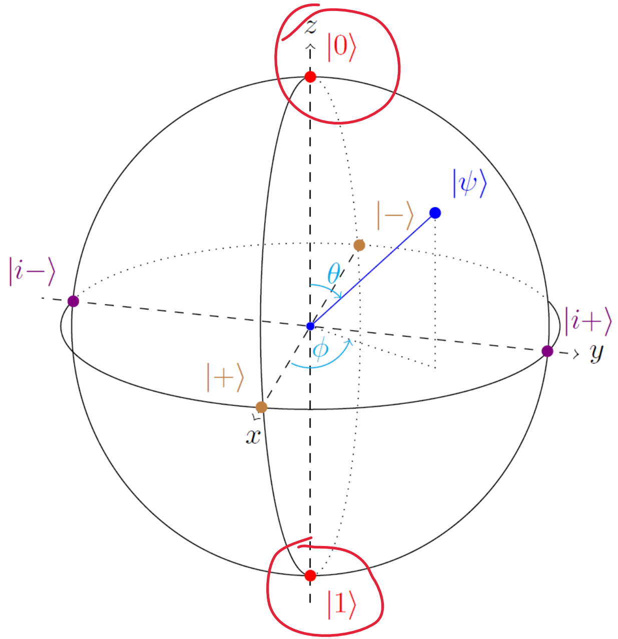


Figure. Bloch sphere.

Qubit

$$\alpha = \frac{1}{\sqrt{2}} \rightarrow |\alpha|^2 = \frac{1}{2}$$

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$\alpha = \cos\frac{\theta}{2}$$

$$\beta = e^{i\phi}\sin\frac{\theta}{2} = (\cos\phi + i\sin\phi)\sin\frac{\theta}{2}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}\,|0\rangle - \frac{1}{\sqrt{2}}\,|1\rangle$$

$$|i+\rangle = \frac{1}{\sqrt{2}}\,|0\rangle + i\frac{1}{\sqrt{2}}\,|1\rangle$$

$$|i-\rangle = \frac{1}{\sqrt{2}}\,|0\rangle - i\frac{1}{\sqrt{2}}\,|1\rangle$$

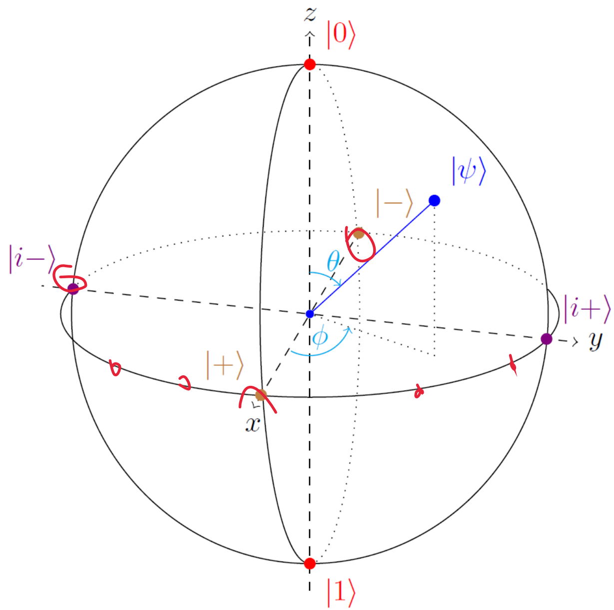


Figure. Bloch sphere.

X = NOT

H = HADAMARD

1-qubit quantum gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

GATE

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \left[|0\rangle\;|1\rangle\right]\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \Rightarrow \begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

$$X\,|\psi\rangle = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}\beta\\ \alpha\end{bmatrix}$$

$$H\,|\psi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\alpha+\beta\\ \alpha-\beta\end{bmatrix}$$

$$H\,|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = |+\rangle$$

$$H\,|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -1\end{bmatrix} = |-\rangle$$

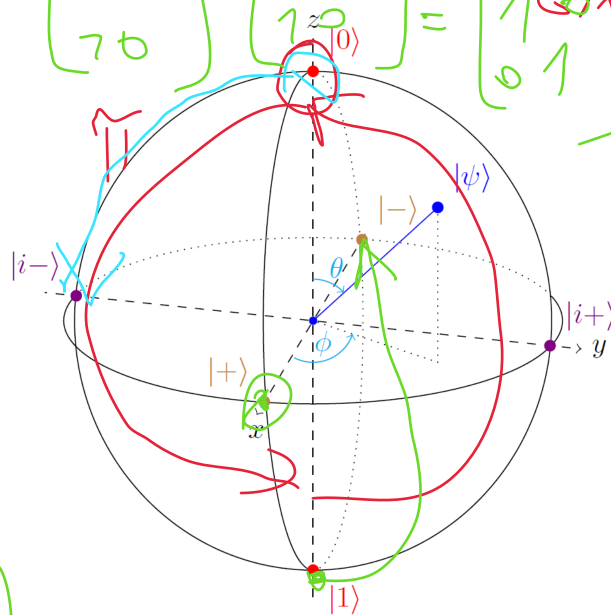


Figure. Bloch sphere.

1-QUBIT QUANTUM GATES

$P(\pi) = Z$ $\quad$ $P\left(\frac{\pi}{4}\right) = T$

$P\left(\frac{\pi}{2}\right) = S$

$$P(\lambda)\left|\psi\right\rangle = \begin{bmatrix}1 & 0\\ 0 & e^{i\lambda}\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}\alpha\\ e^{i\lambda}\beta\end{bmatrix}$$

$$Z\left|\psi\right\rangle = \begin{bmatrix}1 & 0\\ 0 & e^{i\pi}\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}\alpha\\ -\beta\end{bmatrix}$$

$$S\left|\psi\right\rangle = \begin{bmatrix}1 & 0\\ 0 & e^{i\frac{\pi}{2}}\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}\alpha\\ i\beta\end{bmatrix}$$

$$T\left|\psi\right\rangle = \begin{bmatrix}1 & 0\\ 0 & e^{i\frac{\pi}{4}}\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}\alpha\\ e^{i\frac{\pi}{4}}\beta\end{bmatrix} = \begin{bmatrix}\alpha\\ \frac{1}{\sqrt{2}}(1+i)\beta\end{bmatrix}$$

$Z\left|+\right\rangle = \left|-\right\rangle \quad Z\left|-\right\rangle = \left|+\right\rangle \quad S\left|+\right\rangle = \left|i+\right\rangle$

$Z\left|i-\right\rangle = S|S\left|i-\right\rangle = T|T|T|T\left|i-\right\rangle = \left|i+\right\rangle$

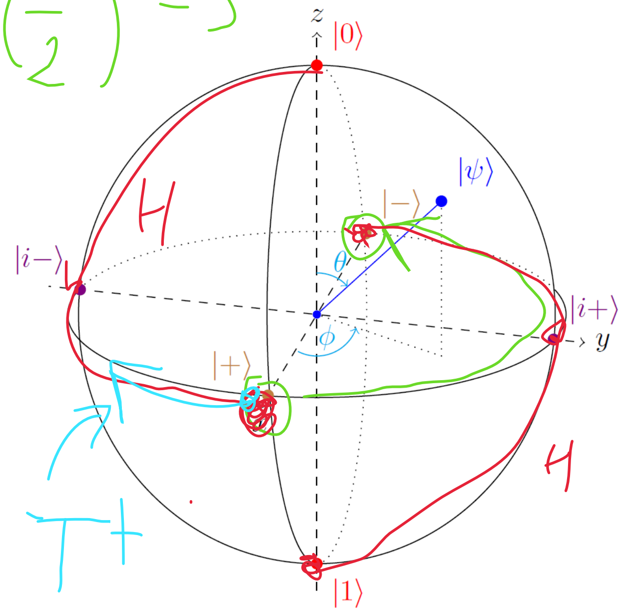
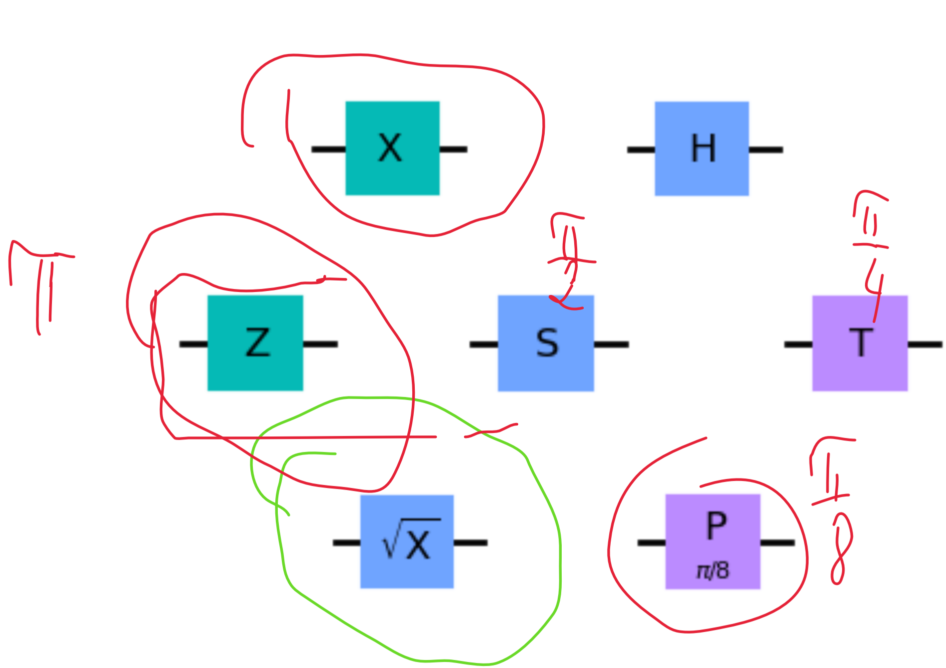


Figure. Bloch sphere.

Implementation in Qiskit



$S^{\dagger}: \; -\frac{\pi}{2}$

$T^{\dagger}: \; -\frac{\pi}{4}$

2-QUBIT QUANTUM GATES

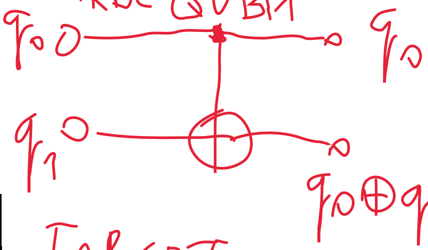
$$\Pr(|00\rangle) = |\alpha_{00}|^2$$

q_1	q_0	q_1
0	0	0
0	1	1
1	0	1
1	1	0

$$|\psi\rangle = \alpha_{00}\,|00\rangle + \alpha_{01}\,|01\rangle + \alpha_{10}\,|10\rangle + \alpha_{11}\,|11\rangle = \big[|00\rangle\ |01\rangle\ |10\rangle\ |11\rangle\big] \begin{bmatrix}\alpha_{00}\\ \alpha_{01}\\ \alpha_{10}\\ \alpha_{11}\end{bmatrix} \Rightarrow \begin{bmatrix}\alpha_{00}\\ \alpha_{01}\\ \alpha_{10}\\ \alpha_{11}\end{bmatrix}$$

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

CONTROL QUBIT



TARGET

$$\text{CNOT} = \boxed{CX}\,|\psi\rangle = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\begin{bmatrix}\alpha_{00}\\ \alpha_{01}\\ \alpha_{10}\\ \alpha_{11}\end{bmatrix} = \begin{bmatrix}\alpha_{00}\\ \alpha_{01}\\ \alpha_{11}\\ \alpha_{10}\end{bmatrix}$$

$$CP(\lambda)\,|\psi\rangle = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&e^{i\lambda}\end{bmatrix}\begin{bmatrix}\alpha_{00}\\ \alpha_{01}\\ \alpha_{10}\\ \alpha_{11}\end{bmatrix} = \begin{bmatrix}\alpha_{00}\\ \alpha_{01}\\ \alpha_{10}\\ e^{i\lambda}\alpha_{11}\end{bmatrix}$$

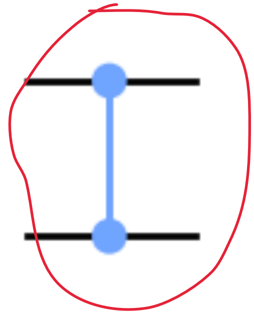
$$SWAP\,|\psi\rangle = \begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\alpha_{00}\\ \alpha_{01}\\ \alpha_{10}\\ \alpha_{11}\end{bmatrix} = \begin{bmatrix}\alpha_{00}\\ \alpha_{10}\\ \alpha_{01}\\ \alpha_{11}\end{bmatrix}$$

Implementation in Qiskit

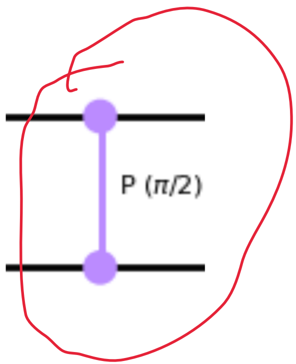
$CZ = CP(\pi)$

CX

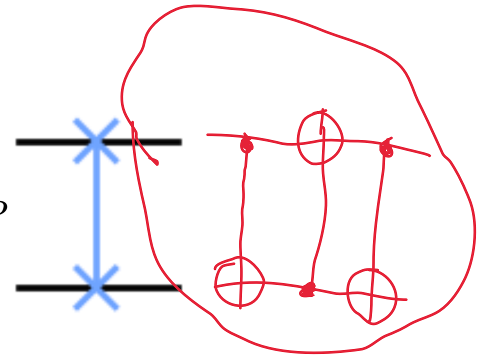
CZ



$CP(\pi/2)$



$SWAP$



Part II

Quantum entanglement

Bell states $= EPR$

$50\% \; |00\rangle$

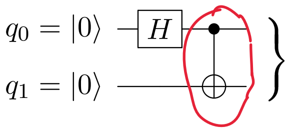
$50\% \; |11\rangle$

$|00\rangle = |0\rangle \otimes |0\rangle$

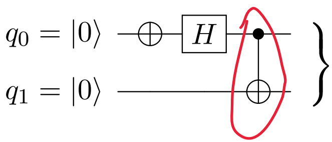
$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$\alpha_{00} = \frac{1}{\sqrt{2}} \quad \alpha_{10} = 0$

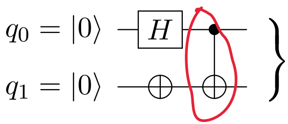
$\alpha_{11} = \frac{1}{\sqrt{2}} \quad \alpha_{01} = 0$



$$|\psi_e\rangle = \boxed{CX}\,\boxed{H}\,|00\rangle = CX\left(\tfrac{1}{\sqrt{2}}\,|00\rangle + \tfrac{1}{\sqrt{2}}\,|01\rangle\right) = \tfrac{1}{\sqrt{2}}\,|00\rangle + \tfrac{1}{\sqrt{2}}\,|11\rangle = |\Phi^+\rangle$$

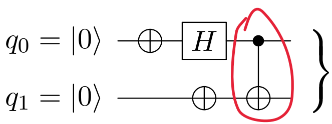


$$|\psi_e\rangle = CX|H\,|00\rangle = CX\left(\tfrac{1}{\sqrt{2}}\,|00\rangle - \tfrac{1}{\sqrt{2}}\,|01\rangle\right) = \tfrac{1}{\sqrt{2}}\,|00\rangle - \tfrac{1}{\sqrt{2}}\,|11\rangle = |\Phi^-\rangle$$



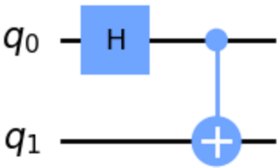
$$|\psi_e\rangle = CX|H\,|00\rangle = CX\left(\tfrac{1}{\sqrt{2}}\,|10\rangle + \tfrac{1}{\sqrt{2}}\,|11\rangle\right) = \tfrac{1}{\sqrt{2}}\,|10\rangle + \tfrac{1}{\sqrt{2}}\,|01\rangle = |\Psi^+\rangle$$

$|\psi^-\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$

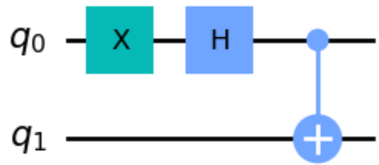


$$|\psi_e\rangle = CX|H\,|00\rangle = CX\left(\tfrac{1}{\sqrt{2}}\,|10\rangle - \tfrac{1}{\sqrt{2}}\,|11\rangle\right) = \tfrac{1}{\sqrt{2}}\,|10\rangle - \tfrac{1}{\sqrt{2}}\,|01\rangle = -|\Psi^-\rangle$$

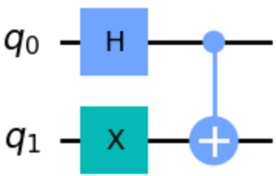
Implementation in Qiskit



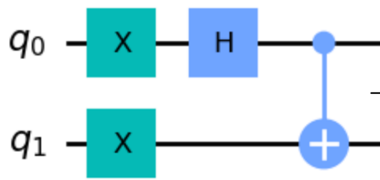
$|\Phi^+\rangle$



$|\Phi^-\rangle$



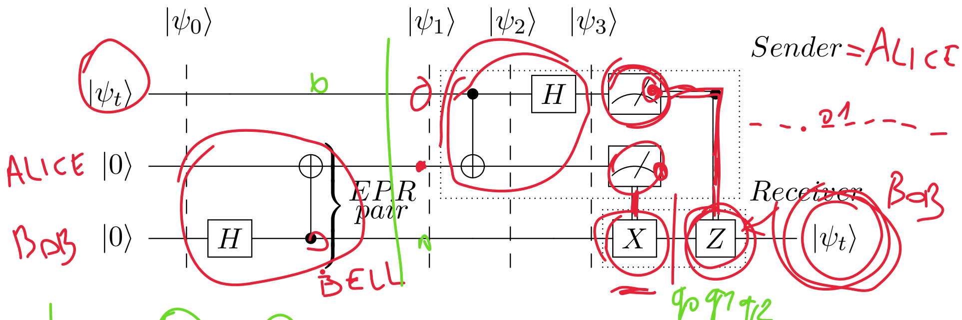
$|\Psi^+\rangle$



$-|\Psi^-\rangle$

Part III

Quantum teleportation



$$|\psi_t\rangle = \alpha_t |0\rangle + \beta_t |1\rangle \quad |\psi_0\rangle = |\psi_t\rangle \otimes |00\rangle = \alpha_t |000\rangle + \beta_t |100\rangle$$

$$|\psi_1\rangle = \frac{\alpha_t}{\sqrt{2}} |000\rangle + \frac{\alpha_t}{\sqrt{2}} |011\rangle + \frac{\beta_t}{\sqrt{2}} |100\rangle + \frac{\beta_t}{\sqrt{2}} |111\rangle$$

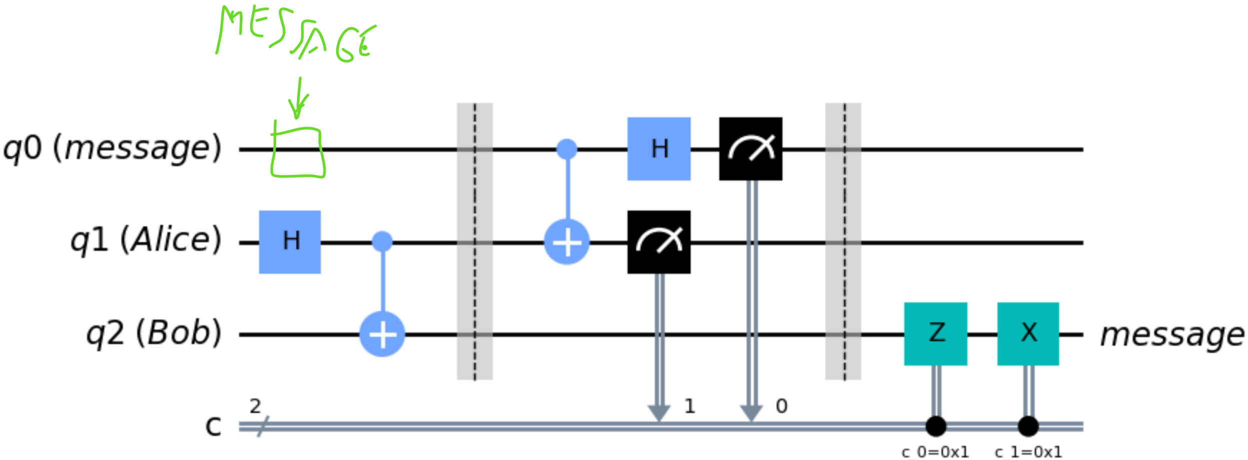
$$|\psi_2\rangle = \frac{\alpha_t}{\sqrt{2}} |000\rangle + \frac{\alpha_t}{\sqrt{2}} |011\rangle + \frac{\beta_t}{\sqrt{2}} |110\rangle + \frac{\beta_t}{\sqrt{2}} |101\rangle$$

$$|\psi_3\rangle = \frac{1}{2} |00\rangle \otimes (\alpha_t |0\rangle + \beta_t |1\rangle) + \frac{1}{2} |01\rangle \otimes (\alpha_t |1\rangle + \beta_t |0\rangle) +$$

$$+ \frac{1}{2} |10\rangle \otimes (\alpha_t |0\rangle - \beta_t |1\rangle) + \frac{1}{2} |11\rangle \otimes (\alpha_t |1\rangle - \beta_t |0\rangle) =$$

$$= \frac{1}{2} |00\rangle \otimes |\psi_t\rangle + \frac{1}{2} |01\rangle \otimes \left|\overline{\psi_t}\right\rangle + \frac{1}{2} |10\rangle \otimes \left|\psi_t^\dagger\right\rangle + \frac{1}{2} |11\rangle \otimes \left|\overline{\psi_t^\dagger}\right\rangle$$

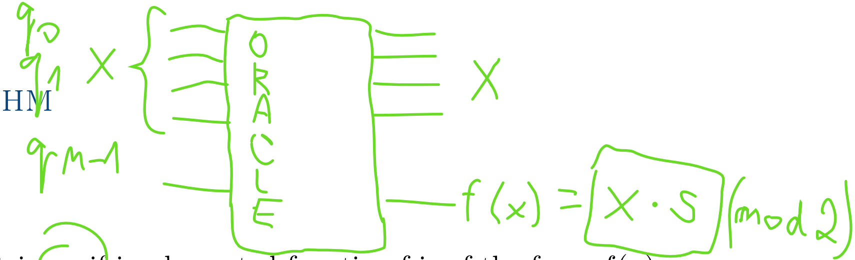
Implementation in Qiskit



Part IV

Bernstein-Vazirani + Deutch-Jozsa algorithm

Bernstein-Vazirani algorithm



$x\cdot s = x_0 s_0 \oplus x_1 s_1 \oplus x_2 s_2 \oplus$
(mod 2)

The problem statement: Find the secret string s if implemented function f is of the form $f(x) = x\cdot s$.

$$|0\rangle^n \xrightarrow{H^{\otimes n}} \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} |x\rangle \xrightarrow{f} \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} (-1)^{f(x)} |x\rangle$$

$$\xrightarrow{H^{\otimes n}} \frac{1}{2^n} \sum_{y\in\{0,1\}^n} \sum_{x\in\{0,1\}^n} (-1)^{f(x)+x\cdot y} |y\rangle = |s\rangle$$

$n=4$

$$f(x)+x\cdot y = x\cdot s + x\cdot y = x\cdot(s\oplus y) = \begin{cases} 0 & (s=y) \\ 0,1,0,1\ldots & (s\neq y) \end{cases}$$

1 2 3 4

0001 0010 0100 1000 $= X$

$\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$

s_0 s_1 s_2 s_3 $= S = S_3 S_2 S_1 S_0$

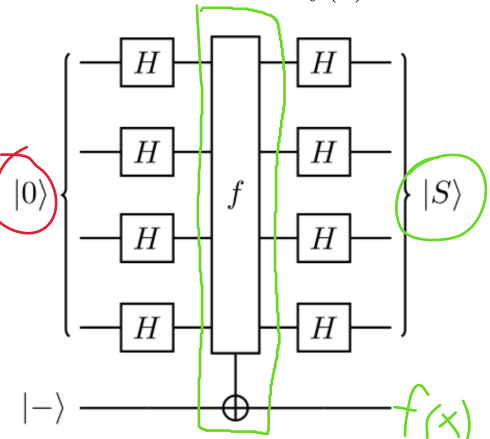


Figure. Bernstein-Vazirani circuit.

Implementation in Qiskit

$s_3 s_2 s_1 s_0$

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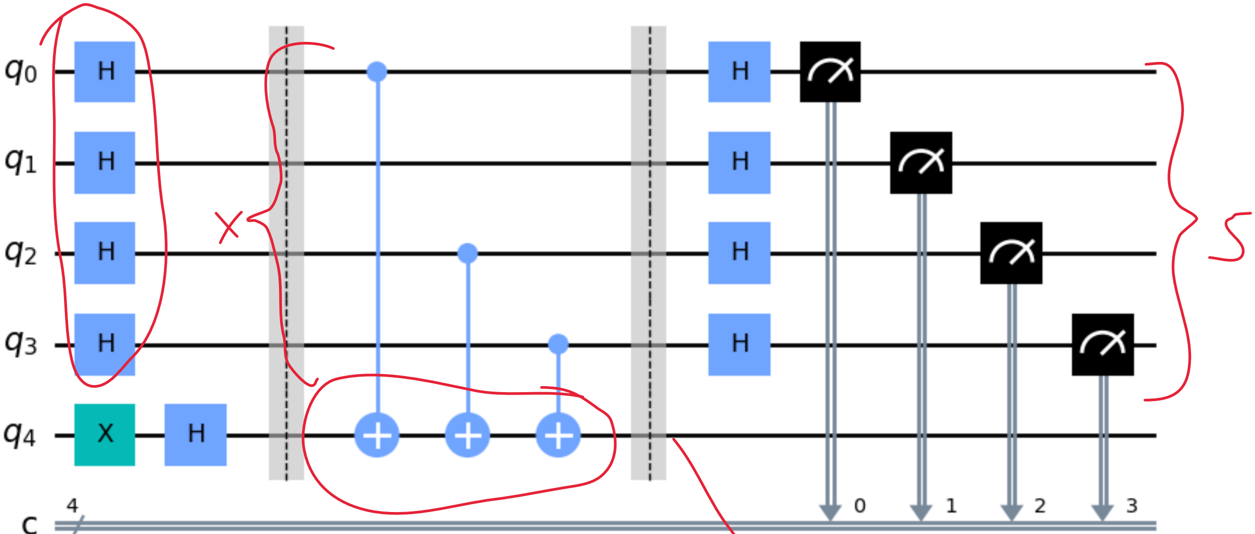


x

s

$f(x) = x \cdot s$

Deutch-Jozsa algorithm

0000 | CONST. | BALANCED
0001 | 0 1 | 0
0010 | 0 1 | 1
⋮ | 0 1 | 0
 | 0 1 | 1
 | ⋮ | 0

$$\frac{2^n}{2}+1 = \underbrace{2^{n-1}+1}$$

<u>The problem statement:</u> Decide whether the implemented function f is constant or balanced.

$$|0\rangle^n \xrightarrow{H^{\otimes n}} \frac{1}{\sqrt{2^n}}\sum_{x\in\{0,1\}^n}|x\rangle \xrightarrow{f} \frac{1}{\sqrt{2^n}}\sum_{x\in\{0,1\}^n}(-1)^{f(x)}|x\rangle$$

$$\xrightarrow{H^{\otimes n}} \frac{1}{2^n}\sum_{y\in\{0,1\}^n}\sum_{x\in\{0,1\}^n}(-1)^{f(x)+x\cdot y}|y\rangle = |s\rangle$$

$$|s\rangle \begin{cases} = 0 \rightarrow \boxed{f \text{ is constant}} \\ \neq 0 \rightarrow f \text{ is balanced} \end{cases}$$

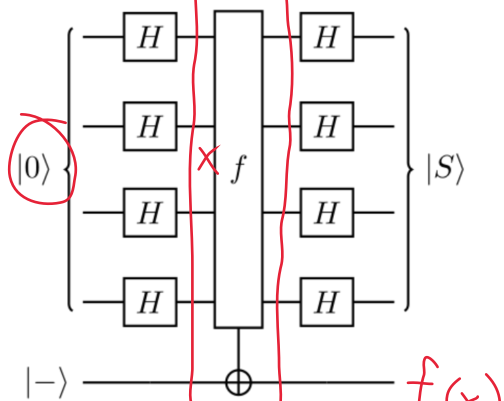
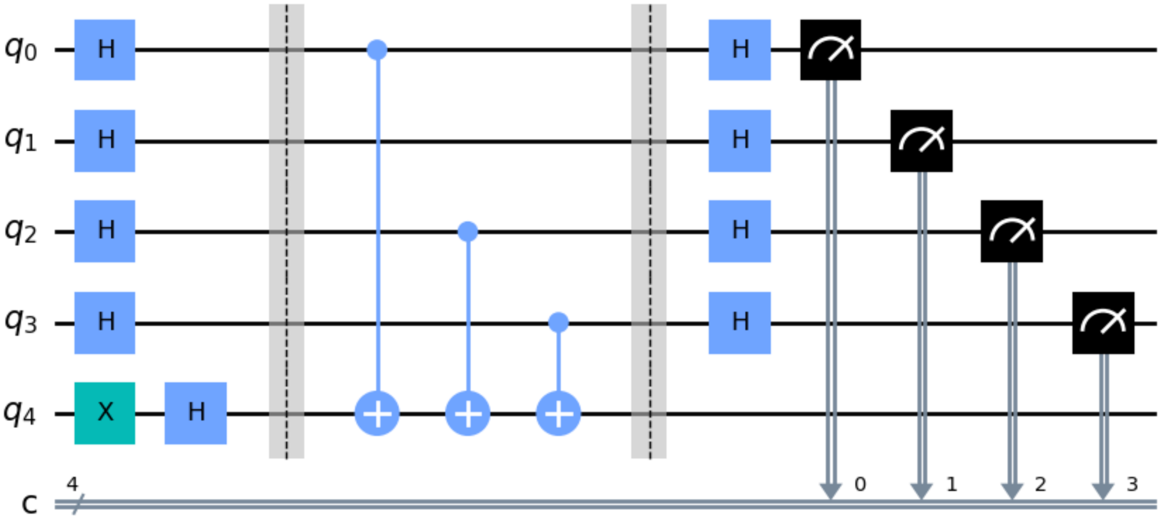


Figure. Deutch-Jozsa circuit.

$$\frac{1}{4}\left(|0000\rangle + |0001\rangle + \quad + |1111\rangle\right)$$

Implementation in Qiskit



Part V

Simon's algorithm

Simon's algorithm

$f : \{0,1\}^n \to \{0,1\}^n$

x		$f(x)$
0000	$\to$	1111
0001	$\to$	1110
=		
1111	$\to$	0000

The problem statement: Decide whether the implemented function f is periodic or not.

$$
|0\rangle^{\otimes n}|0\rangle^{\otimes n} \xrightarrow{H^{\otimes n}} \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} |x\rangle|0\rangle^{\otimes n}
$$

$$
\xrightarrow{U_f} \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} |x\rangle|f(x)\rangle
$$

$$
\xrightarrow{H^{\otimes n}} \frac{1}{2^n} \sum_{y\in\{0,1\}^n} \sum_{x\in\{0,1\}^n} (-1)^{x\cdot y}|y\rangle|f(x)\rangle
$$

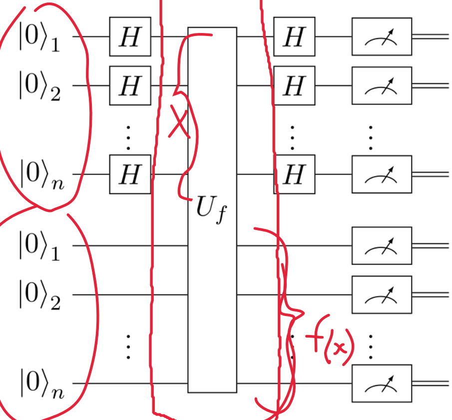


Figure. Simon's circuit.

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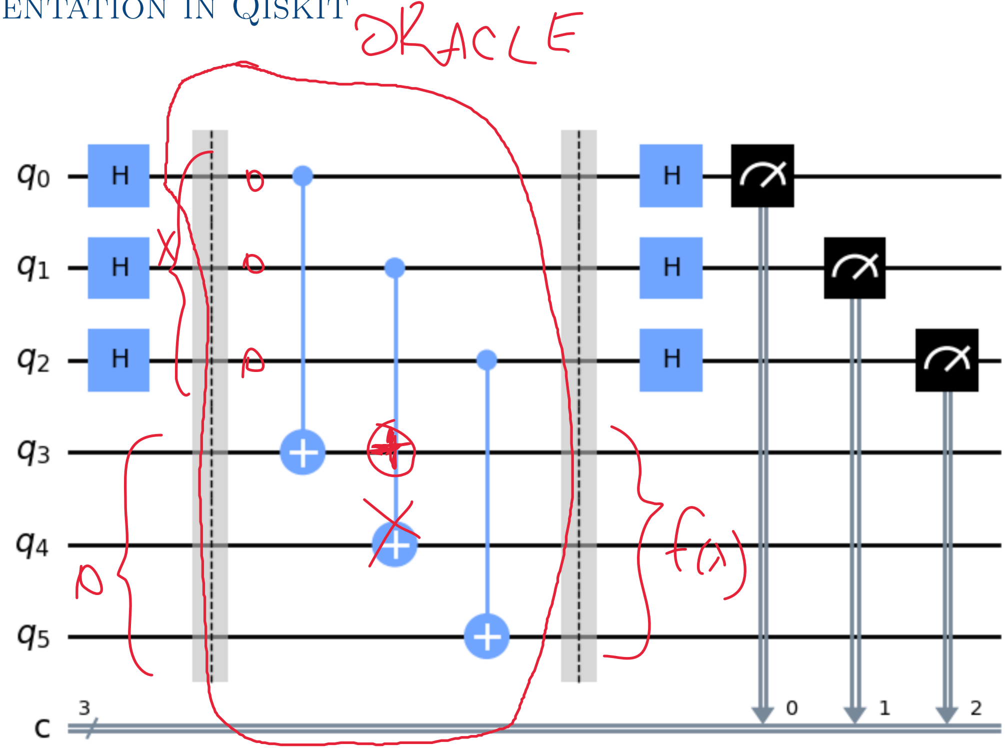
Quantum state after measuring the lower register:

$$
f \text{ is not periodic} \to \frac{1}{\sqrt{2^n}} \sum_{y\in\{0,1\}^n} (-1)^{x_1\cdot y}|y\rangle|f(x_1)\rangle
$$

$$
f \text{ is periodic} \to \frac{1}{\sqrt{2^{n+1+\dots}}} \sum_{y\in\{0,1\}^n} \left[(-1)^{x_1\cdot y} + (-1)^{x_2\cdot y} + \dots\right]|y\rangle|f(x_1)\rangle
$$

$f(x_1) = f(x_2)$

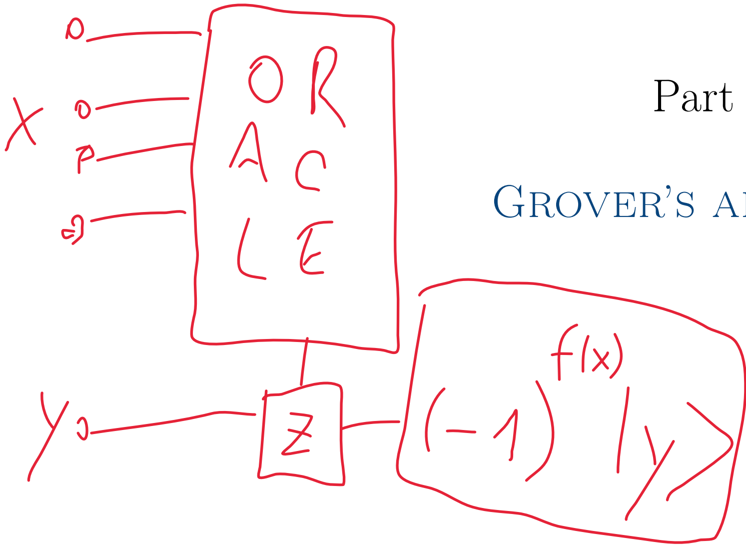
Implementation in Qiskit



Part VI

Grover's algorithm

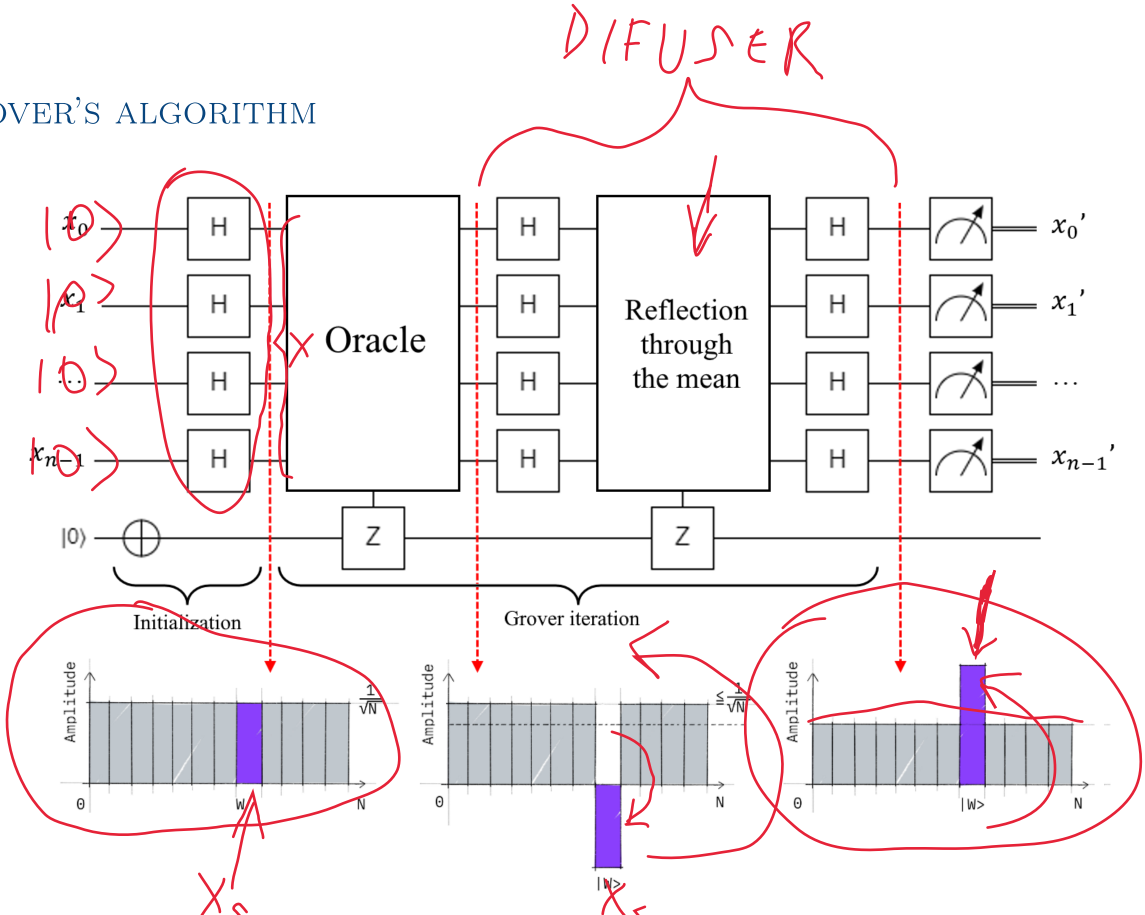
$$f(x) = \begin{cases} 0 & x \neq x_s \\ 1 & x = x_s \end{cases}$$



$$f(x_s) = 1 \Rightarrow x_s = ?$$

$$f(x) = 0 \Rightarrow x \neq x_s$$

Grover's algorithm



Implementation in Qiskit

$Z = HXH$

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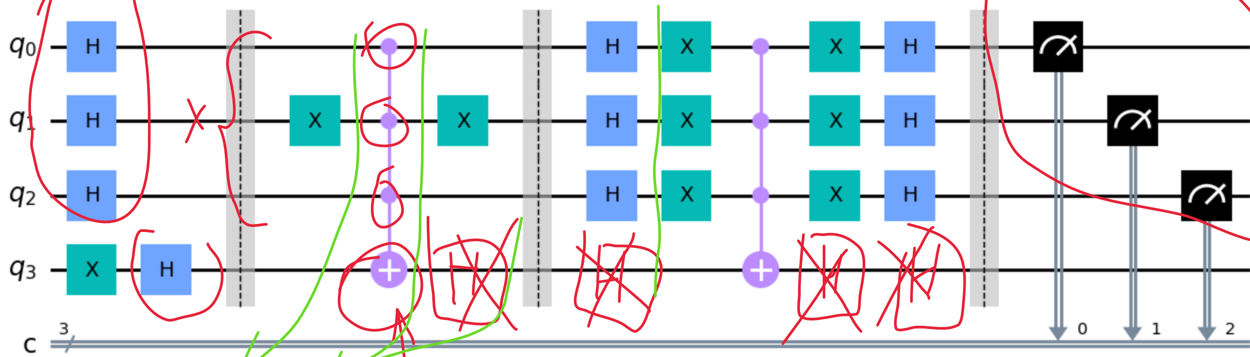
DIFUSER



$x_s = |101\rangle$

$|\psi\rangle = \frac{1}{\sqrt{8}}\left(|000\rangle + |001\rangle + \dots - |101\rangle + |110\rangle + |111\rangle\right)$

Part VII

Quantum Fourier transform

Quantum Fourier transform

$N = 2^n$

n - qubits

$$\text{IDFT: } x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k \cdot e^{2\pi i \frac{kn}{N}}$$

$$\text{QFT}\,|x\rangle = \frac{1}{\sqrt{N}}\sum_{y=0}^{N-1} e^{2\pi i \frac{xy}{N}}\,|y\rangle$$

$$\frac{y}{N} = \frac{y_1 y_2 \dots y_n}{2^n} = \sum_{k=1}^{n}\frac{y_k}{2^k} \longrightarrow \quad \text{QFT}\,|x\rangle = \frac{1}{\sqrt{N}}\sum_{y=0}^{N-1} e^{2\pi i x \sum_{k=1}^{n}\frac{y_k}{2^k}}\,|y_1 y_2 \dots y_n\rangle$$

$$\text{QFT}\,|x\rangle = \frac{1}{\sqrt{2^n}}\sum_{y=0}^{2^n-1}\prod_{k=1}^{2^n} e^{2\pi i x \frac{y_k}{2^k}}\,|y_1 y_2 \dots y_n\rangle$$

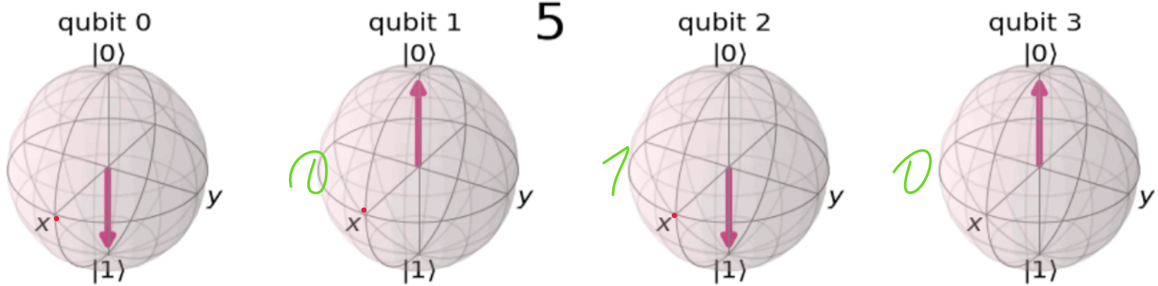
$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\text{QFT}\,|x\rangle = \frac{1}{\sqrt{2^n}}\Big(|0\rangle + e^{i\pi x}\,|1\rangle\Big) \otimes \Big(|0\rangle + e^{i\frac{\pi}{2}x}\,|1\rangle\Big) \otimes \Big(|0\rangle + e^{i\frac{\pi}{4}x}\,|1\rangle\Big) \otimes \cdots\cdots \otimes \Big(|0\rangle + e^{i\frac{\pi}{2^{n-1}}x}\,|1\rangle\Big)$$

π $\quad \frac{\pi}{2}$ $\quad \frac{\pi}{4}$

Quantum Fourier transform

q_3 q_2 q_1 q_0
0 1 0 1

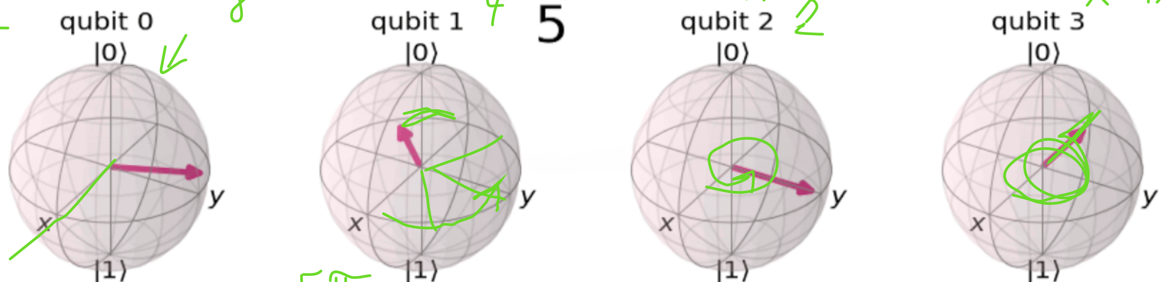


5

$5 \cdot \frac{\pi}{8}$ $x \cdot \frac{\pi}{4}$ $x \cdot \frac{\pi}{2}$ $x \cdot \pi$

$\tilde{5}$

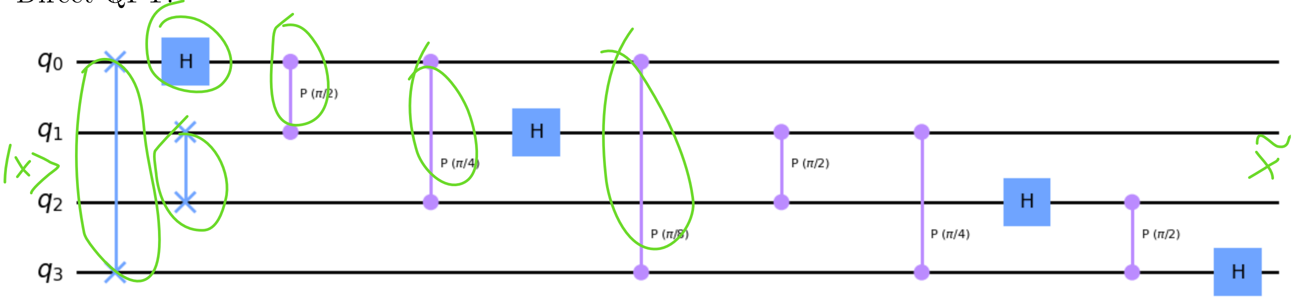
$x \cdot \frac{\pi}{8}$



$\frac{5\pi}{4}$

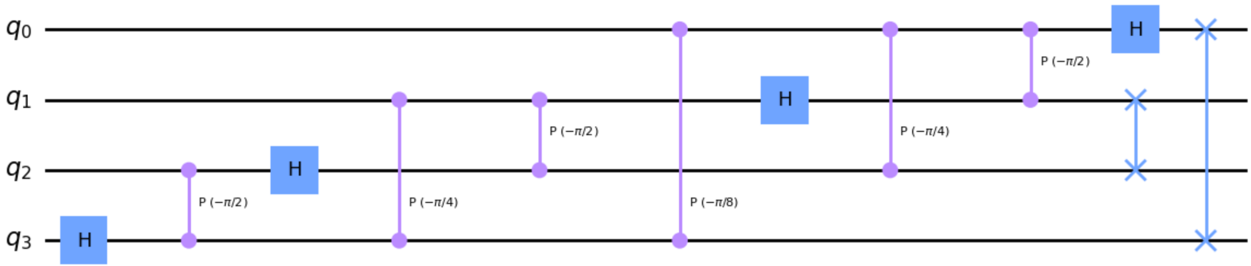
Implementation in Qiskit

Direct QFT:



Implementation in Qiskit

Inverse QFT:



Part VIII

Quantum phase estimation

Quantum phase estimation

Z CZ P($\frac{\pi}{16}$)

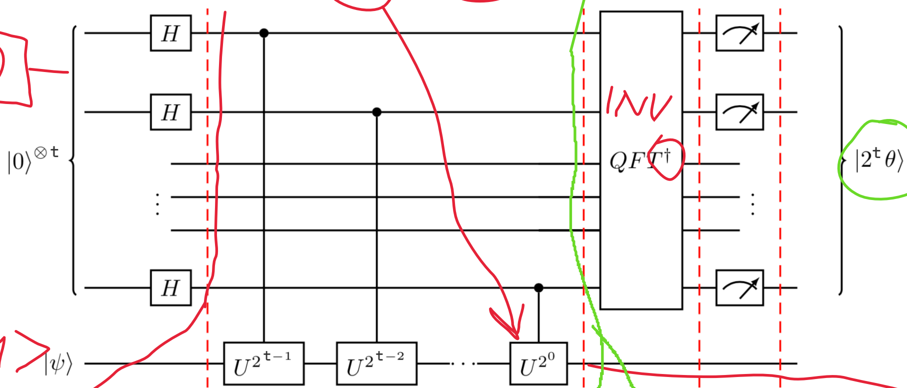
The problem statement:

Estimate the phase of an eigenvalue $e^{2\pi i\theta}$ of a unitary operator U, provided with the corresponding eigenstate ψ:

θ PHASE SHIFT

$$U|\psi\rangle = e^{2\pi i\theta}|\psi\rangle$$

H>



U U

U^2

$|1\rangle$

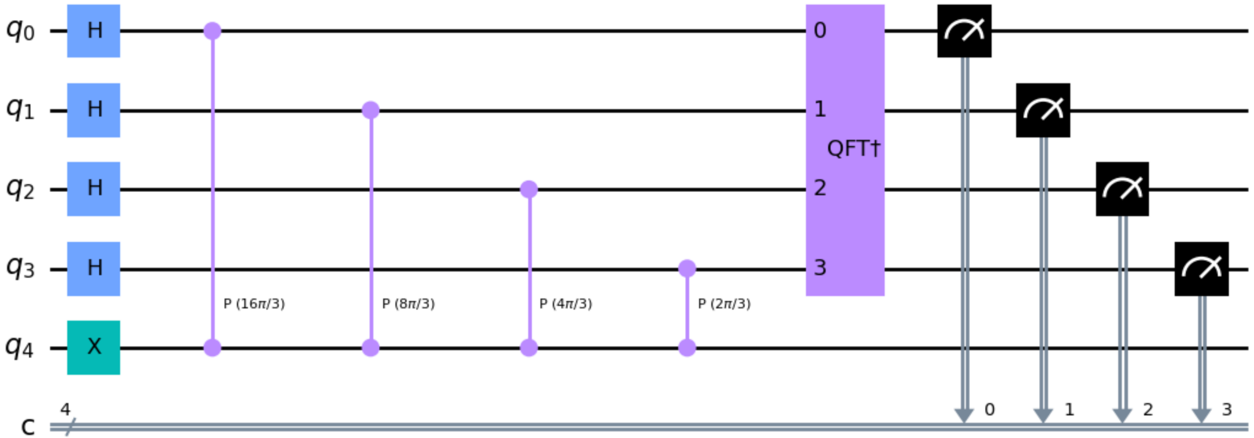
INV

IQFT

$$|0\rangle^{\otimes t} \to \underbrace{\frac{1}{\sqrt{2^t}}\Big(|0\rangle+|1\rangle\Big)^{\otimes t}}_{\varphi=0} \to \frac{1}{\sqrt{2^t}}\Big(|0\rangle+e^{2\pi i\theta 2^{t-1}}|1\rangle\Big)\otimes\Big(|0\rangle+e^{2\pi i\theta 2^{t-2}}|1\rangle\Big)\otimes\cdots\otimes\Big(|0\rangle+e^{2\pi i\theta 2^{0}}|1\rangle\Big) = \text{QFT}\,|2^t\theta\rangle$$

2^{t-1} 2^{t-2}

Implementation in Qiskit



Part IX

Shor's algorithm

Shor's algorithm

$N = P \times R$

$2^{2048} \approx 616$ DIGITS

RSA

The problem statement:

Find factors P, R of number N.

$\gcd(3,15) = 3$

$N = 15$

$\gcd(5,15) = 5$

Shor's algorithm procedure:

1. Pick a random integer number a such that: $1 < a < N$.
2. If $\gcd(a, N) \neq 1$ then $P = a$ and $R = N/a$.
3. Otherwise, find the period r of function $\boxed{f(x) = a^x \bmod N}$.
4. If r is odd then go back to step 1 and choose different a.
5. Otherwise, factors $P, R = \boxed{\gcd(a^{r/2} \pm 1, N)}$. $= \gcd(4 \pm 1, 15)$

$a = 2$

$f(0) = 2^0 \bmod 15 = 1$

$f(x) = 2^x (\bmod 15)$

$f(1) = 2 \bmod 15 = 2$

$f(2) = 4$

$f(3) = 8$

$f(4) = 2^4 \bmod 15 = 1$

$f(5) = 2^5 \bmod 15 = 2$

A quantum computer can be used for step 3, in which it is necessary to create a quantum circuit implementing the modular exponentiation function $f(x) = a^x \bmod N$ and use this circuit instead of the U operator in the quantum phase estimation circuit.
The resulting circuit is called a period-finder circuit and the measured result at the output can then be used to determine the searched period.

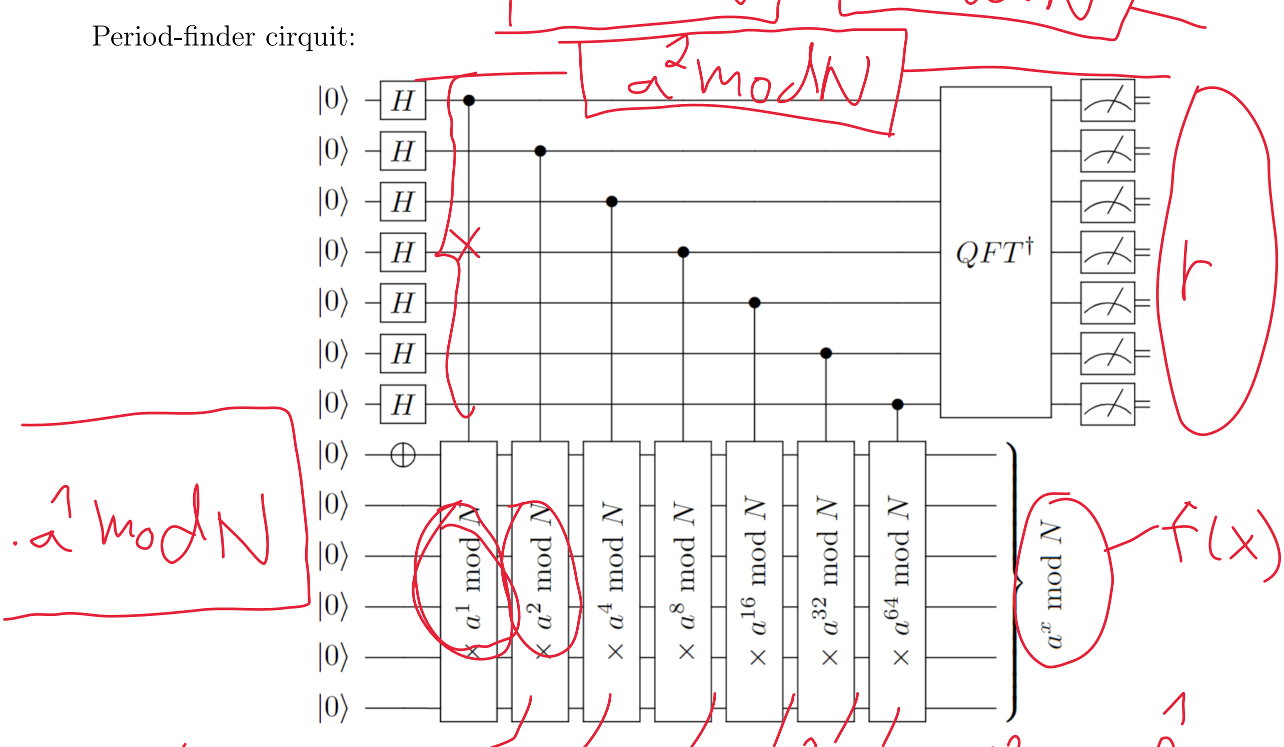
$r = 4$

$f(x) = f(x+4)$

$((y \cdot a^1 \bmod N) \cdot a^1) \bmod N = y a^2 \pmod N$

Shor's algorithm

Period-finder cirquit:



$x = (x_3 x_2 x_1 x_0)_2 = x_3 2^3 + x_2 2^2 + x_1 2^1 + x_0 2^0$

N = 35 a = 6 $f(x) = 6^x \bmod 35$

Implementation in Qiskit

$f(6) = 1$

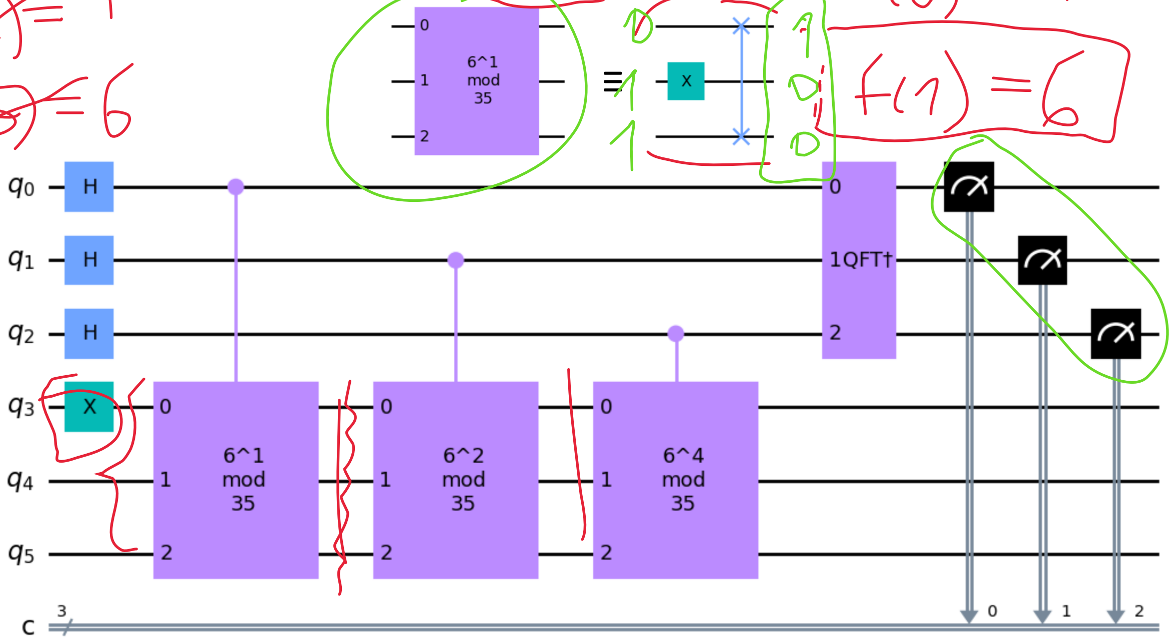
Implementation of the function $g(y) = (y \times 6) \bmod 35$ and below that the overall period-finder circuit designed to find the period of the function $f(x) = 6^x \bmod 35$:

~~$f(2) = 1$~~

~~$f(3) = 6$~~

$f(6) = 1$

$f(1) = 6$



1
0
0

$r = 2 \quad a = 6 \quad N = 35$

$$P, R = \gcd(6^1 \pm 1, 35) = \begin{cases} 7 \\ 5 \end{cases}$$

Thanks

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