Discrimination methods of quantum operators

Paulina Lewandowska

VSB TECHNICAL UNIVERSITY OF OSTRAVA | IT4INNOVATIONS NATIONAL SUPERCOMPUTING CENTER

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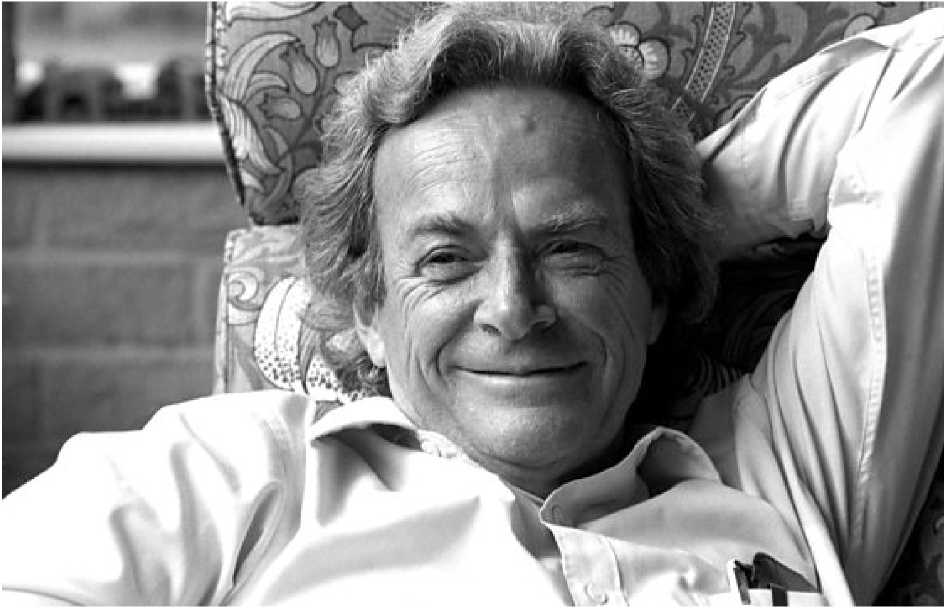
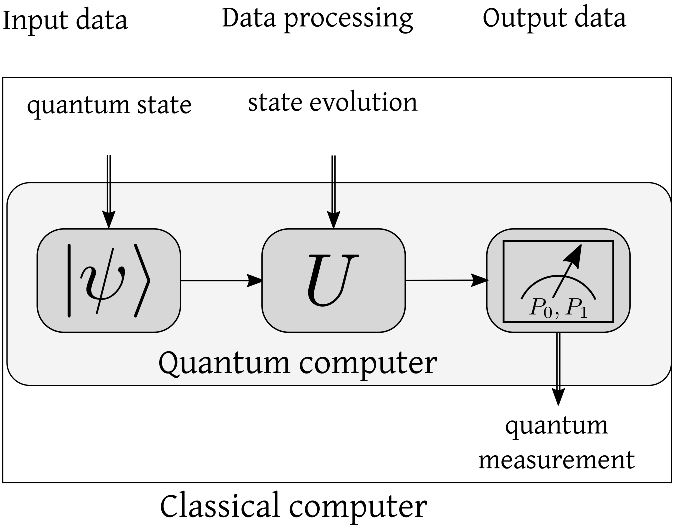
Quantum Computing Lab

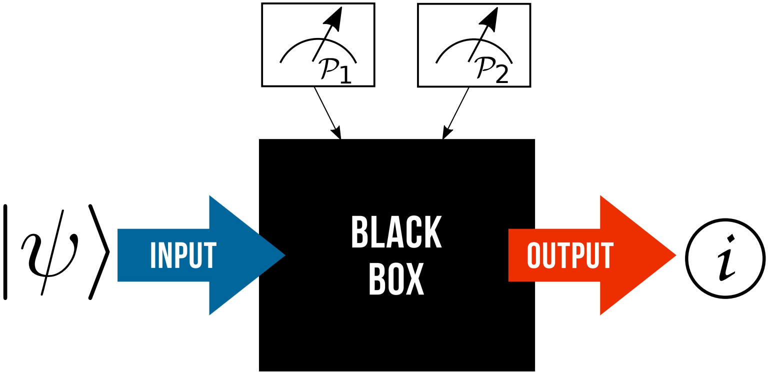
Noisy Intermediate Scale Quantum (NISQ) devices



How does gate-based quantum computer work?



Discrimination task



Notation and mathematical preliminaries

Mathematical preliminaries

Dirac notation

- **ket**: $|\psi\rangle = (x_1, \ldots, x_N)^* \in \mathcal{X}$.
- **bra**: $\langle\psi| = (x_1, \ldots, x_N)^* \in \mathcal{X}^*$.
- **bra-ket**: $\langle\psi|\phi\rangle \in \mathbb{C}$.
- **ket-bra**: $|\psi\rangle\langle\phi|$ – an operator from $\mathrm{L}(\mathcal{X})$ = matrix of size $\dim(\mathcal{X}) \times \dim(\mathcal{X})$.

Quantum state

- **Pure state** – a vector $|\psi\rangle \sim e^{i\alpha}|\psi\rangle \in \mathcal{X}$ normalized $\langle\psi|\psi\rangle = 1$ equivalently a **projective** operator $|\psi\rangle\langle\psi|$.
- **Mixed state** – positive, normalized operator ρ of the form $\rho = \sum_i \alpha_i |\psi_i\rangle\langle\psi_i|$, $\alpha_i \geq 0$ and $\sum_i \alpha_i = 1$.

Set of density operators

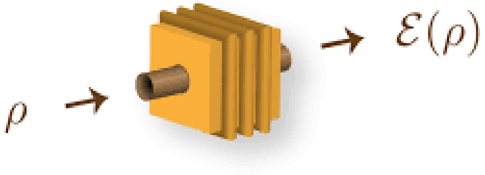
$$\Omega(\mathcal{X}) = \{\rho \in \mathrm{L}(\mathcal{X}) : \rho = \rho^\dagger, \rho \geq 0, \mathrm{Tr}\,\rho = 1\} \quad (1)$$

Mathematical preliminaries

State evolution as a quantum mapping

A quantum channel Φ is a linear mapping which is

- **completely positive**: for every $X \geq 0$ it holds $(\Phi \otimes \mathbb{1})(X) \geq 0$.
- **trace-preserving**: $\mathrm{Tr}(\Phi(X)) = \mathrm{Tr}(X)$ for every X.



Unitary channel

Let $U \in \mathrm{U}(\mathcal{X})$ be a unitary operator. The **unitary channel** Φ_U is defined as

$$\Phi_U(X) = UXU^\dagger. \quad (2)$$

Notation and preliminaries

POVM – Positive Operator Valued Measure

A POVM is a finite collection of positive semidefinite operators $E_i \in \mathrm{Pos}(\mathcal{X})$

$$\mathcal{P} = \{E_1, \ldots, E_m\}$$

such that $\sum_{i=1}^{m} E_i = \mathbb{1}$. The operators E_i are called **effects**.

Figure: Graphical representation of a measurement $\mathcal{P}$.

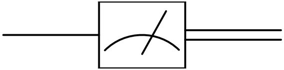


Figure: Graphical representation of a measurement in computational basis.

Born rule

When a quantum state ρ is measured by the quantum measurement $\mathcal{P}$ then the label i is obtained with probability

$$p_i = \mathrm{Tr}(\rho E_i). \tag{3}$$

Von Neumann measurements

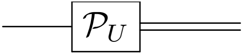
A quantum measurement $\mathcal{P} = \{E_1, \ldots, E_m\}$ is said to be a **projective measurement** if $E_i \in \mathrm{Proj}(\mathcal{X})$ for all $i \in \{1, \ldots, m\}$.

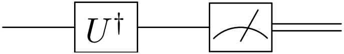
A projective measurement $\mathcal{P} = \{E_1, \ldots, E_m\}$ with rank-one effects E_i is called a **von Neumann measurement**.

Every von Neumann measurement $\mathcal{P}_U$ can be parameterized by $U \in \mathrm{U}(\mathcal{X})$ as

$$\{U |i\rangle\langle i| U^\dagger\}_{i=1}^{\dim(\mathcal{X})}, \tag{4}$$

where $U|i\rangle$ is the i-th column of U.





Numerical range and its properties

Numerical range

For any operator $A \in \mathrm{L}(\mathcal{X})$ one defines its **numerical range** as a subset of complex plane defined by

$$W(A) = \{\langle x| A |x\rangle : |x\rangle \in \mathcal{X}, \langle x|x\rangle = 1\}. \quad (5)$$

In quantum physics the numerical range $W(A)$ contains all possible expectation values of A.

More and more...

https://numericalshadow.org/

Properties of numerical range

Convexity: Hausdorf-Toeplitz theorem

$W(A)$ is a **convex** subset of $\mathbb{C}$.

Compactness

$W(A)$ is a **compact** subset of $\mathbb{C}$.

Numerical range for normal matrices

$W(A)$ contains the spectrum of A.

Numerical range for normal matrices

If $AA^\dagger = A^\dagger A$, then $W(A)$ is **convex hull** of spectrum of A.

Numerical range for Hermitian matrices

If $A = A^\dagger$, then $W(A) = [\lambda_{\min}, \lambda_{\max}]$ forms an **interval** in the real axis.

Examples

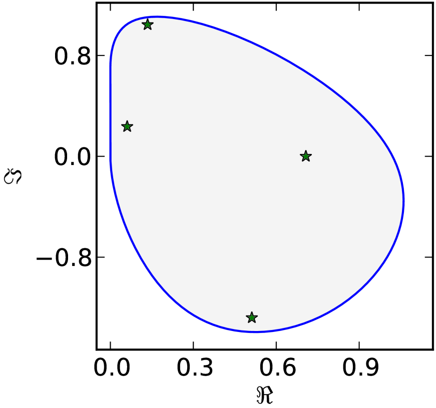


Figure: Numerical range of non-normal matrix $X = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & i & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & i \end{pmatrix}$

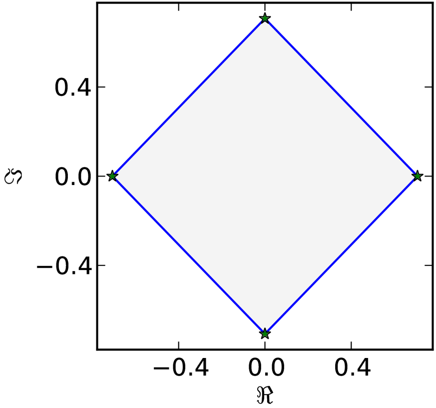
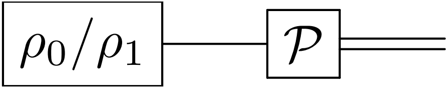


Figure: Numerical range of normal matrix $X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$

Discrimination task – results

Discrimination of quantum states



$$p_{succ}(\rho_0, \rho_1) \coloneqq \max_{\mathcal{P}=\{E_0,E_1\}} \frac{1}{2}\operatorname{Tr}(E_0\rho_0) + \frac{1}{2}\operatorname{Tr}(E_1\rho_1). \tag{6}$$

Holevo-Helstrom theorem

Let $\rho_0, \rho_1 \in \Omega(\mathcal{X})$ be a quantum states. Then, we have

$$p_{succ}(\rho_0, \rho_1) \leq \frac{1}{2} + \frac{1}{4}\|\rho_0 - \rho_1\|_1. \tag{7}$$

Moreover, there exists a projective measurement $\mathcal{P} = \{E_0, E_1\}$ for which Eq. (7) is achieved.

$$\rho_0, \rho_1 \text{ – orthogonal} \implies \|\rho_0 - \rho_1\|_1 = 2 \implies p_{succ}(\rho_0, \rho_1) = 1.$$

Proof of Holevo–Helstrom theorem

1. Define
$$\rho = \frac{1}{2}\rho_0 + \frac{1}{2}\rho_1 \quad \text{and} \quad X = \frac{1}{2}\rho_0 - \frac{1}{2}\rho_1 \tag{8}$$
2. Then
$$\frac{1}{2}\rho_0 = \frac{\rho + X}{2} \quad \text{and} \quad \frac{1}{2}\rho_1 = \frac{\rho - X}{2} \tag{9}$$
3. It implies
$$\frac{1}{2}\operatorname{Tr}(E_0\rho_0) + \frac{1}{2}\operatorname{Tr}(E_1\rho_1) = \frac{1}{2} + \frac{1}{2}\operatorname{Tr}\left((E_0 - E_1)X\right) \tag{10}$$
4. By Holder inequality and $\|E_0 - E_1\| \leq \|E_0 + E_1\| = 1$, we have
$$\begin{aligned} \frac{1}{2} + \frac{1}{2}\operatorname{Tr}\left((E_0 - E_1)X\right) &\leq \frac{1}{2} + \frac{1}{2}\|E_0 - E_1\|\|X\|_1 \leq \frac{1}{2} + \frac{1}{2}\|X\|_1 \\ &= \frac{1}{2} + \frac{1}{4}\|\rho_0 - \rho_1\|_1 \end{aligned} \tag{11}$$

Proof of Holevo–Helstrom theorem

1. To show that equality is achieved for a projective measurement $\{E_0, E_1\}$ one may consider the Jordan–Hahn decomposition

$$X := \frac{1}{2}\rho_0 - \frac{1}{2}\rho_1 = P - Q \tag{12}$$

for $P, Q \geq 0$ such that $PQ = 0$.

2. Define

$$E_0 = \Pi_{\mathrm{im}(P)} \quad \text{and} \quad E_1 = \mathbb{1} - \Pi_{\mathrm{im}(P)} \tag{13}$$

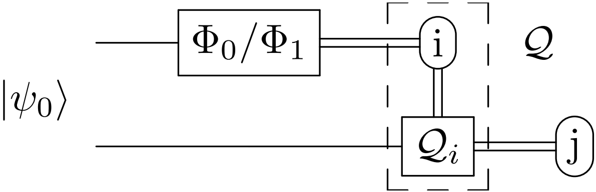
3. Observe that $\{E_0, E_1\}$ is a projective measurement.
4. Calculate

$$\mathrm{Tr}\left((E_0 - E_1)X\right) = \mathrm{Tr}(P) + \mathrm{Tr}(Q) = \|X\|_1 \tag{14}$$

5. Therefore

$$\frac{1}{2}\mathrm{Tr}(E_0\rho_0) + \frac{1}{2}\mathrm{Tr}(E_1\rho_1) = \frac{1}{2} + \frac{1}{2}\|X\|_1 = \frac{1}{2} + \frac{1}{4}\|\rho_0 - \rho_1\|_1 \tag{15}$$

Discrimination of quantum channels



$$p_{succ}(\Phi_0, \Phi_1) \coloneqq \max_{\rho} \max_{\mathcal{Q}=\{E_0,E_1\}} \frac{1}{2} \operatorname{Tr}(E_0(\Phi_0 \otimes \mathbb{1})(\rho)) + \frac{1}{2} \operatorname{Tr}(E_1(\Phi_1 \otimes \mathbb{1})(\rho)) \quad (16)$$

Holevo-Helstrom theorem

Let Φ_0, Φ_1 be quantum channels. Then, we have

$$p_{\text{succ}}(\Phi_0, \Phi_1) = \frac{1}{2} + \frac{1}{4}\|\Phi_0 - \Phi_1\|_\diamond, \quad (17)$$

where $\|\Phi_0 - \Phi_1\|_\diamond = \max_{|\psi\rangle : \||\psi\rangle\|_1 = 1} \| \left((\Phi_0 - \Phi_1) \otimes \mathbb{1}\right) (|\psi\rangle\langle\psi|)\|_1$.

HARD TASK !!!

Primal problem

$$\text{maximize:} \quad \operatorname{Tr}(XM)$$

$$\text{subject to:} \quad \begin{pmatrix} \mathbb{1}_{\mathcal{Y}} \otimes \rho & X \\ X^\dagger & \mathbb{1}_{\mathcal{Y}} \otimes \rho \end{pmatrix} \geq 0,$$

$$\rho \in \Omega(\mathcal{X}),$$

$$X \in \mathrm{L}(\mathcal{X} \otimes \mathcal{Y}).$$

Dual problem

$$\text{minimize:} \quad \| \operatorname{Tr}_{\mathcal{X}}(Y) \|_\infty$$

$$\text{subject to:} \quad \begin{pmatrix} Y & -M \\ -M & Y \end{pmatrix} \geq 0,$$

$$Y \in \mathrm{Pos}(\mathcal{X} \otimes \mathcal{Y}).$$

Table: Formulation of primal and dual problem for calculating the diamond norm of the Hermiticity-preserving map with Choi operator M.

Discrimination of unitary channels

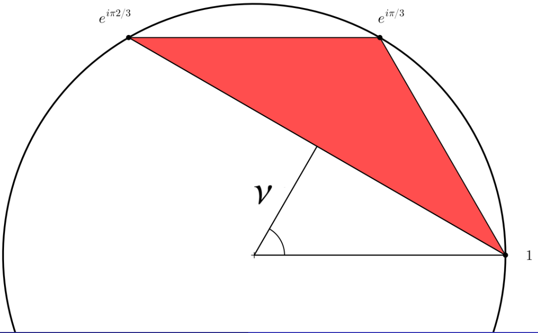
Let $\Phi_U, \Phi_{\mathbb{1}}$ be two unitary channels. Then,

$$\|\Phi_U - \Phi_{\mathbb{1}}\|_\diamond = 2\sqrt{1-\nu^2},$$

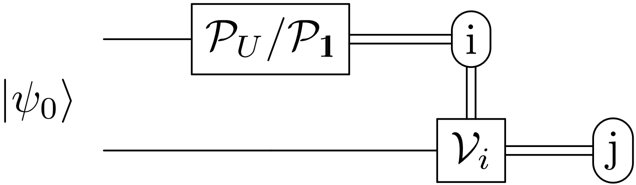
where $\nu = \{|x| : x \in W(U^\dagger)\}$.

Observe if $0 \in W(U^\dagger) \implies \Phi_U, \Phi_{\mathbb{1}}$ – perfectly distinguishable

Consider $U^\dagger$ with eigenvalues $1, \mathrm{e}^{i\pi/3}, \mathrm{e}^{2i\pi/3}$.
Then, the figure represents the numerical range $W\left(U^\dagger\right)$.



Discrimination of von Neumann measurements



$$p_{\text{succ}}(\mathcal{P}_U, \mathcal{P}_{\mathbb{1}}) = \frac{1}{2} + \frac{1}{4} \|\mathcal{P}_U - \mathcal{P}_{\mathbb{1}}\|_\diamond,$$

where $\|\mathcal{P}_U - \mathcal{P}_{\mathbb{1}}\|_\diamond = \max\limits_{|\psi\rangle : \||\psi\rangle\|_1 = 1} \| \left((\mathcal{P}_U - \mathcal{P}_{\mathbb{1}}) \otimes \mathbb{1} \right) (|\psi\rangle\langle\psi|) \|_1 .$

Diamond norm for von Neumann measurements

$$\|\mathcal{P}_U - \mathcal{P}_{\mathbb{1}}\|_\diamond = \min_{E \in \mathrm{DU}(\mathcal{X})} \|\Phi_{UE} - \Phi_{\mathbb{1}}\|_\diamond, \tag{18}$$

where $\|\Phi_U - \Phi_{\mathbb{1}}\|_\diamond = 2\sqrt{1-\nu^2}$, and $\nu(U) = \min_{x \in W(U)} |x|$.

Puchała, Z., Pawela, Ł., Krawiec, A., Kukulski, R. *Strategies for optimal single-shot discrimination of quantum measurements*. Physical Review A, 98(4), 042103, (2018).

Calculating the optimal probability

$$\min_{E\in\mathrm{DU}(\mathcal{X})} \|\Phi_{UE} - \Phi_{\mathbb{1}}\|_\diamond = \min_{E\in\mathrm{DU}(\mathcal{X})} 2\sqrt{1-\nu^2(UE)} = \min_{E\in\mathrm{DU}(\mathcal{X})} 2\sqrt{1 - \min_{\rho\in\Omega(\mathcal{X})} |\operatorname{Tr}\rho UE|^2}$$
$$= 2\sqrt{1 - \max_{E\in\mathrm{DU}(\mathcal{X})} \min_{\rho\in\Omega(\mathcal{X})} |\operatorname{Tr}\rho UE|^2}$$

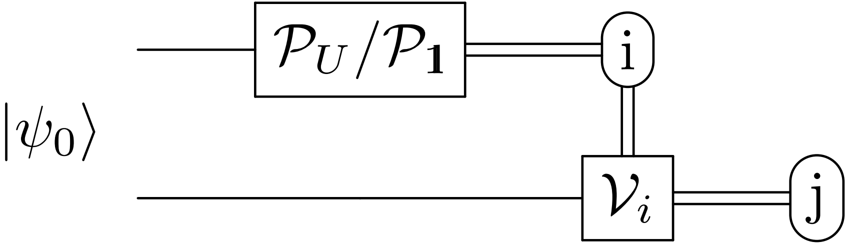
$$\max_{E\in\mathrm{DU}(\mathcal{X})} \min_{\rho\in\Omega(\mathcal{X})} \left|\operatorname{Tr}\rho UE\right|^2 = \min_{\rho\in\Omega(\mathcal{X})} \max_{E\in\mathrm{DU}(\mathcal{X})} \left|\operatorname{Tr}\rho UE\right|^2$$
$$= \min_{\rho\in\Omega(\mathcal{X})} \sum_i |\langle i|\rho U|i\rangle|$$

Primal problem

$$\text{minimize:} \quad \|\mathrm{diag}(U^\dagger \rho)\|_1$$
$$\text{subject to:} \quad \rho \in \Omega(\mathcal{X}).$$

SDP for calculation the distance of von Neumann measurements $\|\mathcal{P}_U - \mathcal{P}_{\mathbb{1}}\|_\diamond$.

Components of the optimal strategy



Optimal initial state

1. Assume that $E_0 \in \mathrm{DU}(\mathcal{X})$ satisfies the condition
$$||\Phi_{UE_0} - \Phi_{\mathbb{1}}||_\diamond = ||\mathcal{P}_U - \mathcal{P}_{\mathbb{1}}||_\diamond < 2. \tag{19}$$
There exist states $\rho_1, \rho_d \in \Omega(\mathcal{X})$, such that
$$\begin{aligned} \rho_1 &= \Pi_1 \rho_1 \Pi_1, \\ \rho_d &= \Pi_d \rho_d \Pi_d, \\ \mathrm{diag}(\rho_1) &= \mathrm{diag}(\rho_d), \end{aligned} \tag{20}$$
where Π_1 and Π_d be the projectors onto the subspaces spanned by the eigenvectors λ_1 and λ_d where λ_1, λ_d be a pair of the most distant eigenvalues of UE_0. Furthermore, the discriminator $|\psi_0\rangle = \left|\sqrt{\rho^\top}\right\rangle\!\rangle$, where ρ is defined as
$$\rho = \frac{1}{2}\rho_1 + \frac{1}{2}\rho_d. \tag{21}$$
2. $\mathcal{P}_U$ and $\mathcal{P}_{\mathbb{1}}$ are perfectly distinguishable if and only if there exists $\rho \in \Omega(\mathcal{X})$ such that
$$\mathrm{diag}(U^\dagger \rho) = 0. \tag{22}$$
Moreover, the quantum state $\left|\sqrt{\rho^\top}\right\rangle\!\rangle$ is a discriminator.

Geometrical interpretation

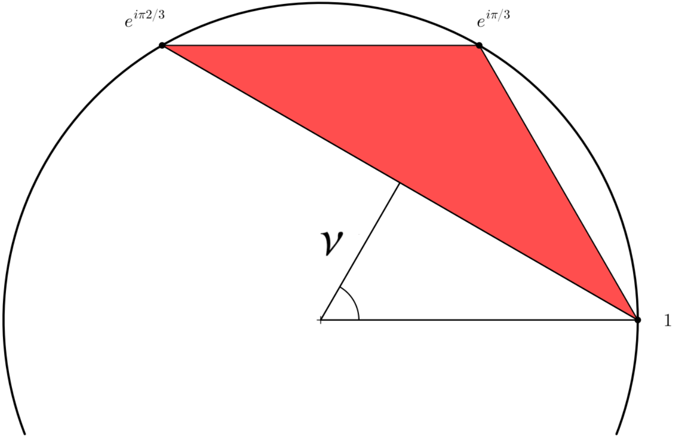


Figure: The graphical representation of $W\left((UE_0)^\dagger\right)$. Let us take the most distant eigenvalues of $(UE_0)^\dagger$ and create $\rho = \frac{1}{2}\Pi_{\min} + \frac{1}{2}\Pi_{\max}$ and the discriminator has the form $|\psi_0\rangle = \left|\sqrt{\rho^\top}\right\rangle\!\!\Big\rangle$.

Optimal final measurement

1. Let $|\psi_0\rangle$ be the discriminator and let
$$X = (\mathcal{P}_U \otimes \mathcal{I}_{\mathcal{X}})(|\psi_0\rangle\langle\psi_0|) - (\mathcal{P}_{\mathbb{1}} \otimes \mathcal{I}_{\mathcal{X}})(|\psi_0\rangle\langle\psi_0|), \tag{23}$$
2. From Hahn-Jordan decomposition, we write
$$X = P - Q, \tag{24}$$
where $P, Q \geq 0$ such that $PQ = 0$.
3. We create projectors $\Pi_{\mathrm{im}(P)}$ and $\Pi_{\mathrm{im}(Q)}$ by
$$\Pi_{\mathrm{im}(P)} = \sum_{i=0}^{d-1} |i\rangle\langle i| \otimes \Pi_{\mathrm{im}(P),i}, \tag{25}$$
and
$$\Pi_{\mathrm{im}(Q)} = \sum_{i=0}^{d-1} |i\rangle\langle i| \otimes \Pi_{\mathrm{im}(Q),i}, \tag{26}$$
where $\Pi_{\mathrm{im}(P),i}, \Pi_{\mathrm{im}(Q),i} \in \mathrm{Proj}(\mathcal{X})$ are orthogonal projectors.
4. For $i \in \{0, \ldots, d-1\}$, we define $\mathcal{V}_i$ by
$$\mathcal{V}_i = \left\{\Pi_{\mathrm{im}(P),i}, \Pi_{\mathrm{im}(Q),i}\right\}. \tag{27}$$

PyQBench – an innovative Python library for benchmarking gate-based quantum computers

PYQBENCH

```
pip install pyqbench
```

More and more...

Github: `https://github.com/iitis/PyQBench`
Documentation: `https://pyqbench.readthedocs.io/en/latest/`

Some comments about PyQBench

- PyQBench benchmarks based on the scheme of discrimination for any **qubit** von Neumann measurements.
- PyQBench allows the user to test various architectures, available through `qiskit` and Amazon BraKet `qiskit-braket-provider`.
- PyQBench offers a simplified, ready-to-use, **command line interface** (CLI) for running benchmarks using a predefined parametrized Fourier family of measurements.
- For more advanced scenarios, PyQBench offers a way of employing user-defined measurements as a **Python library**.

Discrimination scheme for parameterized family of Fourier measurements

The parametrized family of Fourier measurements $\{\mathcal{P}_{U_\phi} : \phi \in [0, 2\pi]\}$, where

$$U_\phi = H \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} H^\dagger, \tag{28}$$

and H is the Hadamard matrix od dimension two.

- Optimal initial state (discriminator)

$$|\psi_0\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle\right). \tag{29}$$

- The optimal final measurements $\mathcal{Q}_0$ and $\mathcal{Q}_1$ are von Neumann measurements defined as $\mathcal{Q}_i = \mathcal{P}_{V_i}$, where

$$V_0 = \begin{pmatrix} i\sin\left(\frac{\pi-\phi}{4}\right) & -i\cos\left(\frac{\pi-\phi}{4}\right) \\ \cos\left(\frac{\pi-\phi}{4}\right) & \sin\left(\frac{\pi-\phi}{4}\right) \end{pmatrix}, V_1 = V_0 \cdot X \tag{30}$$

- $$p_{\text{succ}}(\mathcal{P}_{U_\phi}, \mathcal{P}_{\mathbb{1}}) = \frac{1}{2} + \frac{|1 - e^{i\phi}|}{4}. \tag{31}$$

Implementation of von Neumann measurement

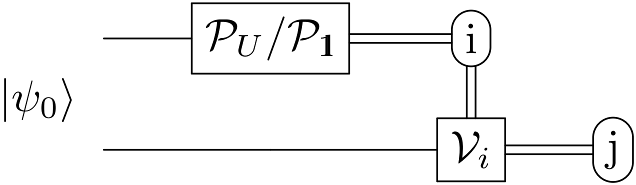


Figure: Scheme of discrimination of von Neumann measurements $\mathcal{P}_U$ and $\mathcal{P}_{\mathbb{1}}$.

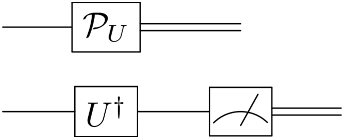
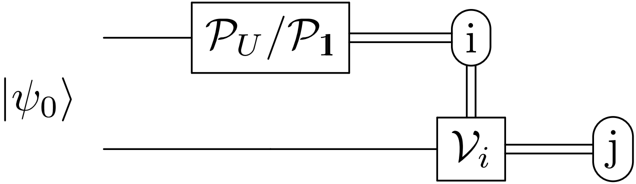


Figure: Implementation von Neumann measurement $\mathcal{P}_U$ on quantum computer.

Implementation of controlled measurement



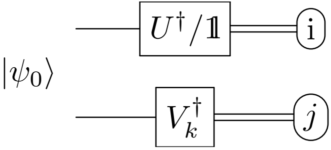


Figure: Scheme of discrimination $\mathcal{P}_U$ and $\mathcal{P}_{\mathbb{1}}$ using postselection.

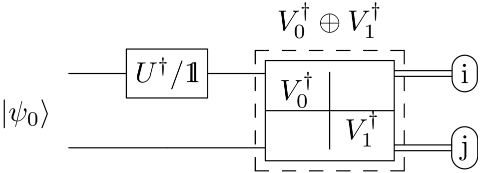
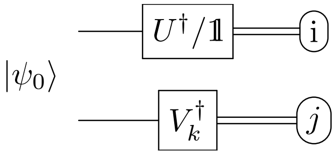


Figure: Scheme of discrimination $\mathcal{P}_U$ and $\mathcal{P}_{\mathbb{1}}$ using direct sum $V_0^\dagger \oplus V_1^\dagger$.

Postselection



$$\{(\mathcal{P}, k, i, j) : \mathcal{P} = \{\mathcal{P}_U, \mathcal{P}_{\mathbb{1}}\}, i \in \{0,1\}, j \in \{0,1\}, k \in \{0,1\}\}$$

We discard all the experiments for which $i \neq k$. The total number of valid experiments is:

$$N_{\text{total}} = \#\{(\mathcal{Q}, k, i, j) : k = i\}. \tag{32}$$

If we define

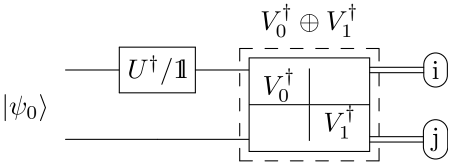
$$N_{\mathcal{P}_U} = \#\{(\mathcal{Q}, k, i, j) : \mathcal{Q} = \mathcal{P}_U, k = i, j = 0\}, \tag{33}$$

$$N_{\mathcal{P}_{\mathbb{1}}} = \#\{(\mathcal{Q}, k, i, j) : \mathcal{Q} = \mathcal{P}_{\mathbb{1}}, k = i, j = 1\}, \tag{34}$$

then the empirical success probability can be computed as

$$p_{\text{succ}}(\mathcal{P}_U, \mathcal{P}_{\mathbb{1}}) = \frac{N_{\mathcal{P}_U} + N_{\mathcal{P}_{\mathbb{1}}}}{N_{\text{total}}}. \tag{35}$$

Direct sum



$$\{(\mathcal{P}, i, j) : \mathcal{P} = \{\mathcal{P}_U, \mathcal{P}_{\mathbb{1}}\}, i \in \{0,1\}, j \in \{0,1\}\}$$

The number of successful trials for U and $\mathbb{1}$, respectively, can be written as

$$N_{\mathcal{P}_U} = \#\{(\mathcal{Q}, i, j) : \mathcal{Q} = \mathcal{P}_U, j = 0\}, \quad (36)$$

$$N_{\mathcal{P}_{\mathbb{1}}} = \#\{(\mathcal{Q}, i, j) : \mathcal{Q} = \mathcal{P}_{\mathbb{1}}, j = 1\}. \quad (37)$$

Then, the probability of correct discrimination between $\mathcal{P}_U$ and $\mathcal{P}_{\mathbb{1}}$ is given by

$$p_{\text{succ}} = \frac{N_{\mathcal{P}_U} + N_{\mathcal{P}_{\mathbb{1}}}}{N_{\text{total}}}, \quad (38)$$

where N_{total} is the number of trials.

Workflow of CLI

1. Preparing configuration files describing the backend and the experiment scenario.
2. Submitting/running experiments. Depending on the experiment scenario, execution can be synchronous, or asynchronous.
3. (optional) Checking the status of the submitted jobs if the execution is asynchronous.
4. Resolving asynchronous jobs into the actual measurement outcomes.
5. Converting obtained measurement outcomes into tabulated form.

Defining the experiment file and backend (YML)

```yaml
type: discrimination-fourier
qubits:
   - target: 0
     ancilla: 1
   - target: 1
     ancilla: 2
   - target: 14
     ancilla: 16
angles:
   start: 0
   stop: 2 * pi
   num_steps: 32
gateset: ibmq
method: direct_sum
num_shots: 8192
```

```yaml
name: ibmq_kolkata
asynchronous: false
provider:
   hub: ibm-q-psnc
   group: open
   project: main
```

Figure: Defining the experiment file and backend (YML).

Steps

```
qbench <benchmark-type> <command> <parameters>
```

Figure: Command syntax of qbench.

```
qbench disc-fourier benchmark experiment_file.yml backend_file.yml
    --output sync_results.yml
```

Results using synchronous mode


```
data:
- target: 0
  ancilla: 1
  phi: 0.0
  results_per_circuit:
  - name: id
  histogram: {'00': 28, '01': 26, '10': 21, '11': 25}
  mitigation_info:
   target: {prob_meas0_prep1: 0.05220000000000024,
       prob_meas1_prep0: 0.0172}
   ancilla: {prob_meas0_prep1: 0.0590000000000005,
       prob_meas1_prep0: 0.0202}
  mitigated_histogram: {'00': 0.2637212373658018, '01':
      0.25865061319892463, '10': 0.2067279352110304, '11':
      0.2709002142242433}
```

Results from asynchronous mode


```
metadata:
  experiments:
    type: discrimination-fourier
    qubits:
    - {target: 0, ancilla: 1}
    - {target: 1, ancilla: 2}
    angles: {start: 0.0, stop: 6.283185307179586, num_steps: 3}
    gateset: ibmq
    method: direct_sum
    num_shots: 100
  backend_description:
    name: ibmq_quito
    asynchronous: true
    provider: {group: open, hub: ibm-q, project: main}
data:
- job_id: 63e7f17a17b7ed49ca24e05b
  keys:
  - [0, 1, id, 0.0]
  - [0, 1, u, 0.0]
  - [0, 1, id, 3.141592653589793]
  - [0, 1, u, 3.141592653589793]
```

Steps

```
qbench <benchmark-type> <command> <parameters>
```

Figure: Invocation of qbench script.

```
qbench disc-fourier benchmark experiment_file.yml backend_file.yml
    --output sync_results.yml
```

```
qbench disc-fourier status async_results.yml
```

```
qbench disc-fourier resolve async-results.yml resolved.yml
```

```
qbench disc-fourier tabulate results.yml results.csv
```

The resulting CSV file

target	ancilla	phi	ideal_prob	disc_prob	mit_disc_prob
0	1	0	0.5	0.503	0.503
0	1	0.202	0.550	0.542	0.544
0	1	0.405	0.602	0.598	0.602
0	1	0.608	0.647	0.636	0.639
0	1	0.811	0.697	0.679	0.684
0	1	1.013	0.743	0.726	0.731
0	1	1.216	0.786	0.769	0.775
0	1	1.419	0.826	0.803	0.810
0	1	1.621	0.862	0.843	0.851
0	1	1.824	0.895	0.873	0.882

Results

IMB Q Kolkata with 27 qubits

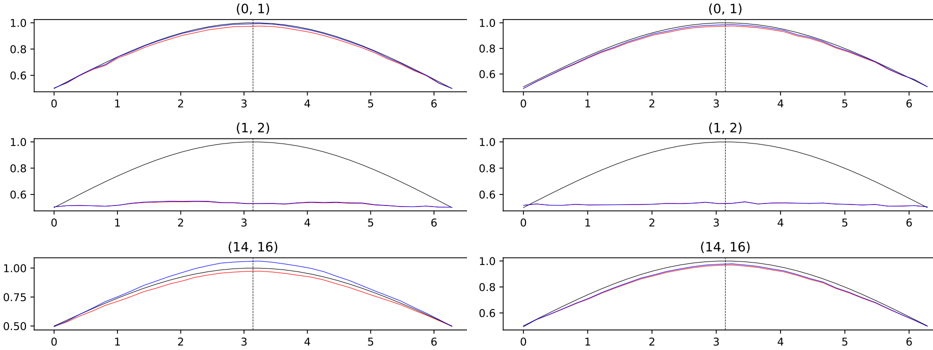


Figure: Discrimination experiment defined for the parameterized family of von Neumann measurement using postselection (left figure) and direct sum (right figure). The theoretical probability is given by black line. Red line represents the empirical probability. The blue line represents the empirical probability after using the package Mthree error mitigation.

Open question

How to implement discrimination scheme for parametrized family of Fourier measurements of dimension d?

$$U_\phi = F_d \begin{pmatrix} \phi_1 & 0 & \cdots & 0 \\ 0 & \phi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi_d \end{pmatrix} F_d$$

Thank you for your attention!