USING SYMMETRIES

TO PREPARE GROUND STATES ON QUANTUM COMPUTERS

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THE CHALLENGE: EXPONENTIAL HILBERT SPACE

N spin-1/2 particles → Hilbert space dimension **2^N**

2^{12}

12 spins
4 096 states

2^{30}

30 spins
1 073 741 824 states

2^{50}

50 spins
1 125 899 906 842 624 states

Classical computers hit a wall. Quantum computers can help, and they help much more when they are guided to the relevant part of this space.

KEY QUESTION: HOW DO WE FOCUS THE ALGORITHM ON THE PART OF THE HILBERT SPACE THAT ACTUALLY MATTERS?

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Symmetries reduce the relevant Hilbert space, increase the spectral gap, and give quantum algorithms a cleaner problem to solve, leading to faster and more reliable convergence.

OVERVIEW

1

PHYSICAL MODEL

Spin Hamiltonian inspired by the neutrinos

2

SYMMETRIES & HILBERT SPACE

Decomposition, spectral gap

3

PROJECTION ALGORITHM

Preparing the initial quantum state

4

RESULTS

Comparison of performance with and without symmetries

1

PHYSICAL MODEL

Spin Hamiltonian inspired by the neutrinos

THE NEUTRINO-INSPIRED MODEL

All-to-All Random Couplings

$$H = \sum_{i<j} \left(1 - \cos\theta_{ij}\right)\ \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

(all-to-all connections)

12 spins on a **4×3 lattice** with all-to-all random interactions
Full Hilbert space: 2^{12} = **4096** dimensions
Coupling coefficient (1 − cos θ_{ij}): cos θ_{ij} is randomly chosen from the range [-1, 1]
Symmetries:

- total spin J,
- z-projection Jz

2

SYMMETRIES AND HILBERT SPACE

Decomposition, spectral gap

SYMMETRY AS HILBERT SPACE DECOMPOSITION

When H commutes with a symmetry operator → Hilbert space decomposes into independent subspaces, each labelled by a quantum number (e.g. total spin J).

- States in different subspaces never mix under time evolution.
- If the ground state has J = 0, we restrict the entire algorithm to that subspace and ignore everything else.

KEY INSIGHT: WE DO NOT NEED TO EXPLORE THE FULL HILBERT SPACE — SYMMETRY TELLS US EXACTLY WHERE TO LOOK.

SYMMETRY AS HILBERT SPACE DECOMPOSITION

4 096

Full space

→

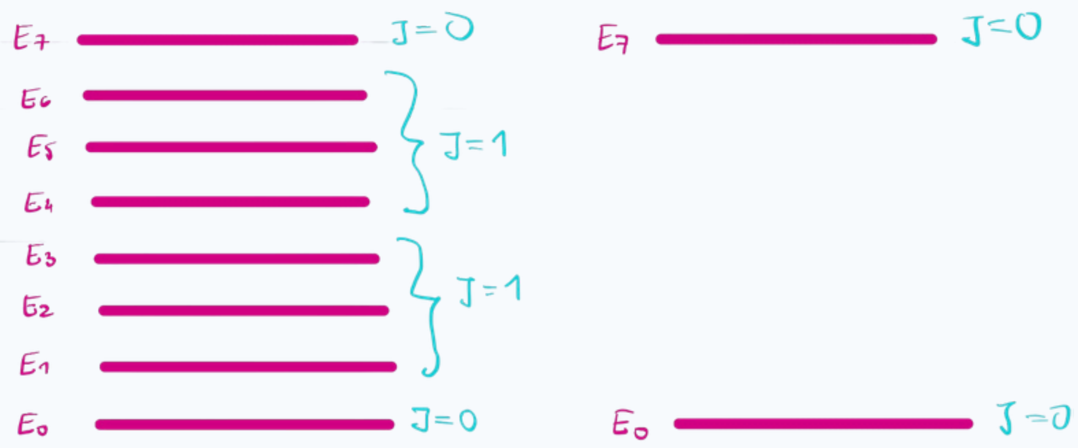
924

Project **Jz = 0**

→

132

Restrict **J = 0**



Symmetry gives us: smaller space, larger gap and more focused algorithm.

3

INITIAL STATES AND PROJECTION ALGORITHM

Preparing the initial quantum state

STARTING POINT — PRODUCT STATE

- Simple product state: tensor product of single-qubit states, easy to prepare with single-qubit gates only
- **Neutrino model:** flavor-momentum locked state; each spin aligned to its momentum vector (semiclassical ground state for large N)

$$|\psi_i\rangle = \bigotimes_{i=1}^{N} \exp\left(i\pi \boldsymbol{k_i}\cdot\frac{\boldsymbol{\sigma}_i}{2}\right)|0\rangle_i\,,$$

- This state capture some physics but do not respect quantum symmetries

MODEL	INITIAL FIDELITY
Neutrino model	2.4%

Fidelity = $|\langle\psi\,|\,\varphi\rangle|^2$

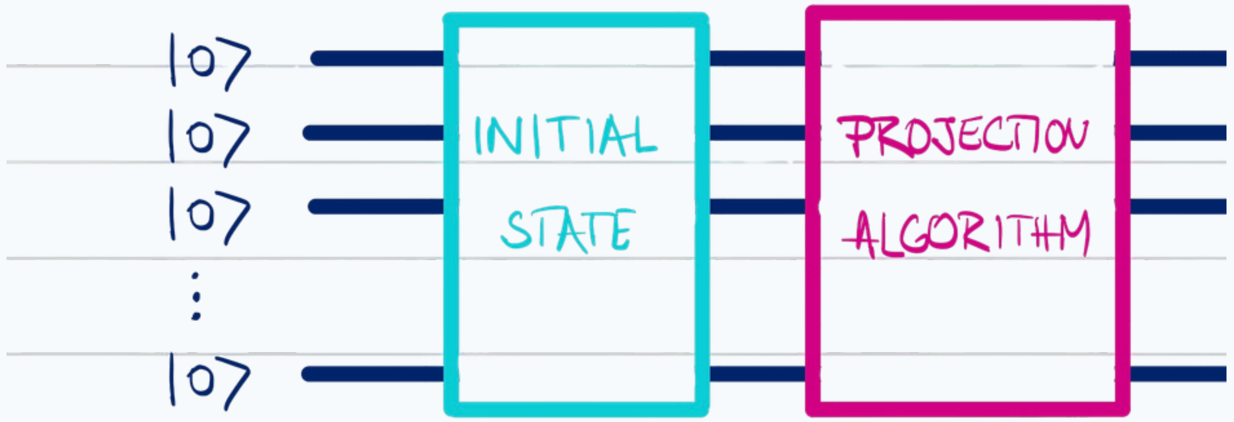
ψ = prepared state φ = exact ground state

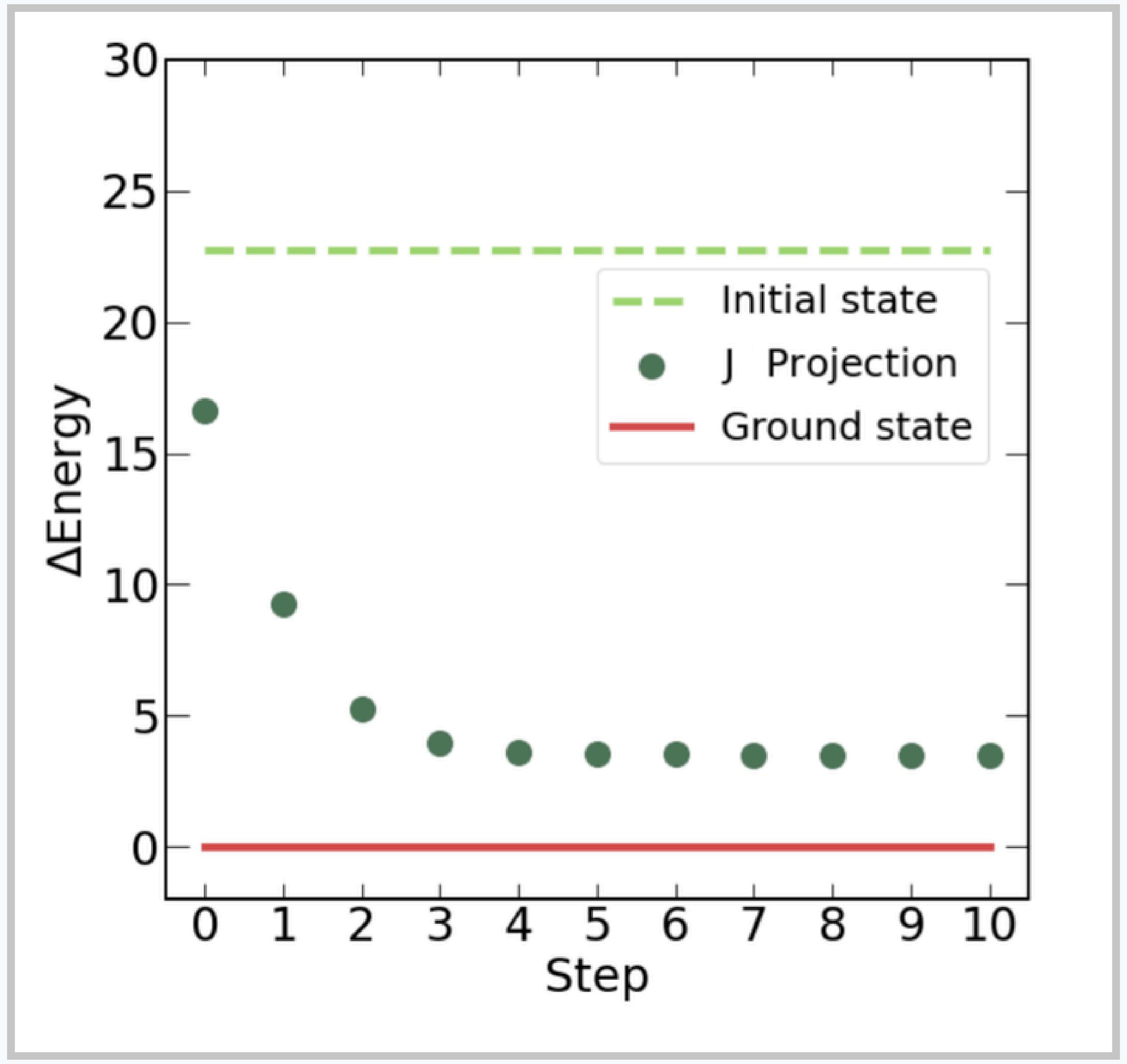
Low initial fidelity but a good warm start for the projection step.

THE PROJECTION ALGORITHM

Alternating projections onto
Jz = 0 and Jx = 0, each followed by
a measurement of the ancilla, iteratively
filter out the J > 0 components. 11 steps
are enough to reach 56.7% fidelity.

$$|\psi(t_i)\rangle = e^{-it_i J_z \otimes Y_a}\ |\psi\rangle \otimes |0\rangle_a$$
$$|\psi(t_i)\rangle = e^{-it_i J_x \otimes Y_a}\ |\psi\rangle \otimes |0\rangle_a$$





~80% of the way to exact ground state after 11 steps

After projection: fidelity **2.4% → 56.7%**

4

RESULTS

Comparison of performance with and without symmetries

VARIATIONAL QUANTUM EIGENSOLVER

PREPARE ANSATZ $U(\Theta)$

Parameterized quantum circuit

MEASURE ENERGY

$\langle\psi(\theta)|H|\psi(\theta)\rangle$

CLASSICAL OPTIMIZER

Update θ to minimize energy

Repeat until convergence

- **Variational principle:** $\langle\psi|H|\psi\rangle \geq E_0$ for any trial state, minimizing energy gives best approximation
- **Main challenge:** barren plateaus; gradient becomes exponentially small, optimizer gets stuck
- **Solutions:** start close to the correct answer, reduce number of parameters
- Symmetry-projected initial state is an ideal warm start; fidelity already **57%**

VQE: WITHOUT VS. WITH SYMMETRY PROJECTION

Without Projection

- Searches full 4096-dimensional Hilbert space
- No guidance on which subspace matters
- Prone to barren plateaus and local minima
- May converge to wrong symmetry sector
- Low final fidelity with exact ground state

With Projection

- Searches only the correct symmetry subspace
- Starts from high-fidelity projected state
- Better-behaved optimization landscape
- Swap ansatz preserves J and Jz throughout
- Fidelity: >98%

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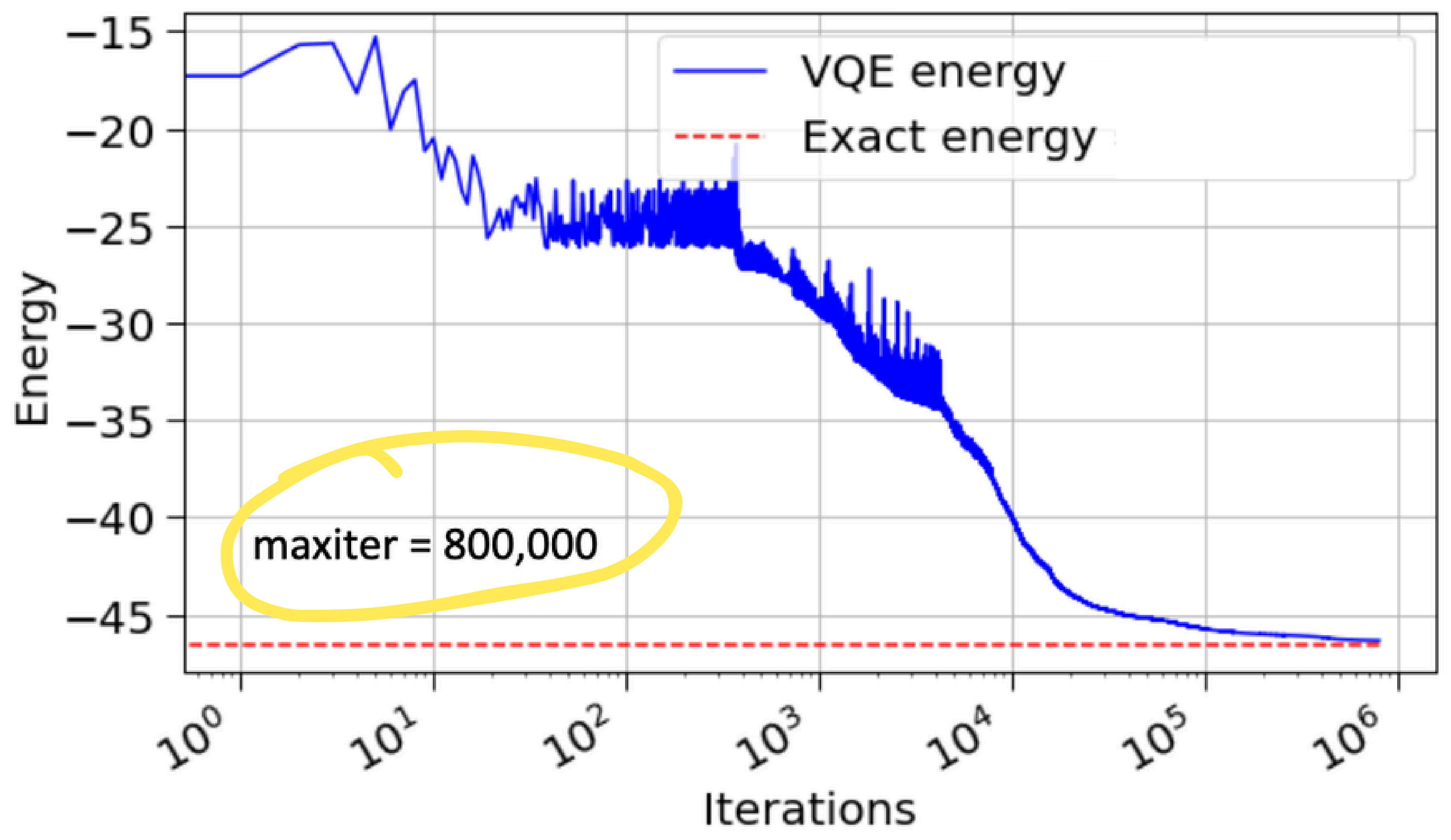
Without Projection



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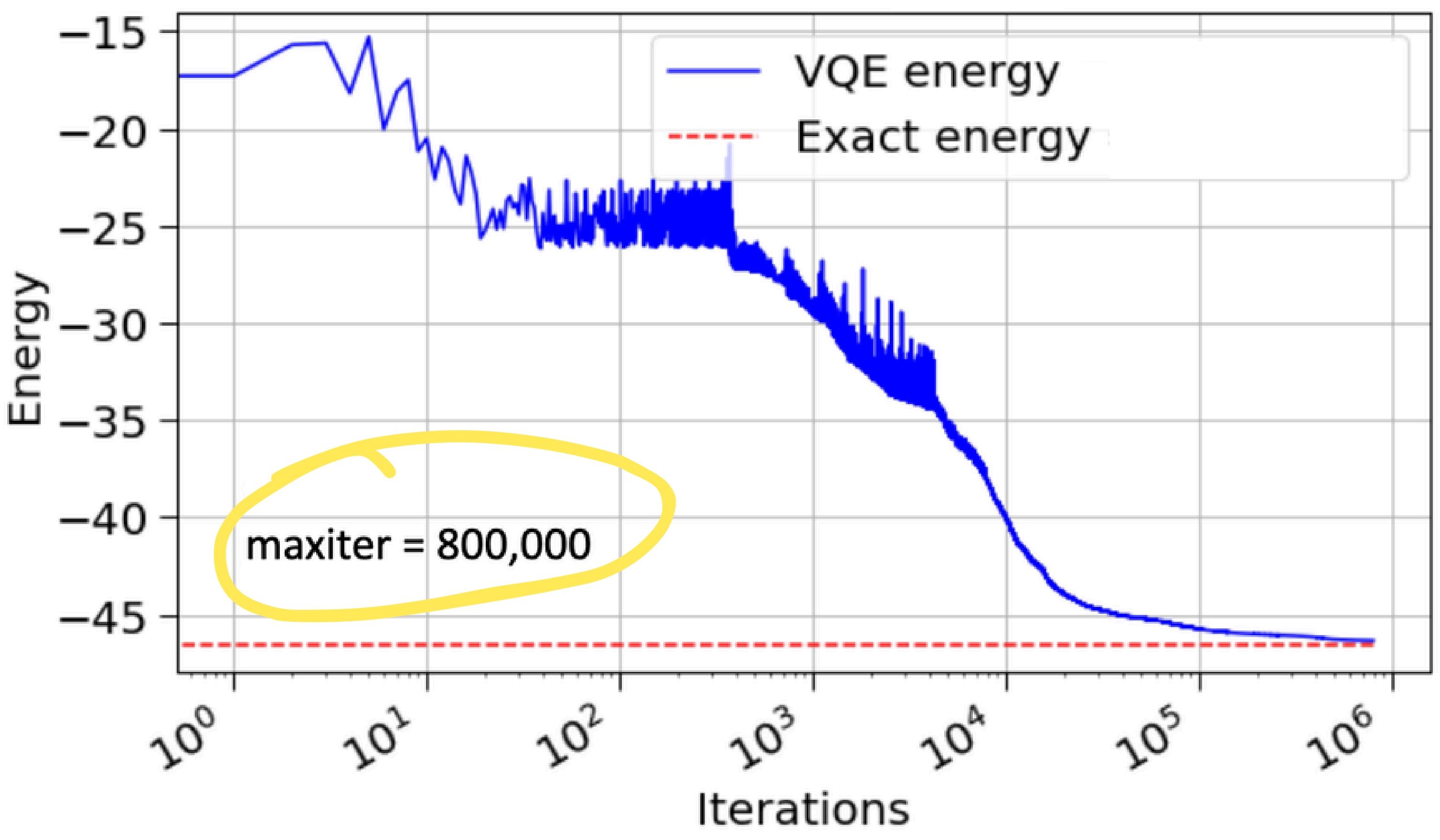
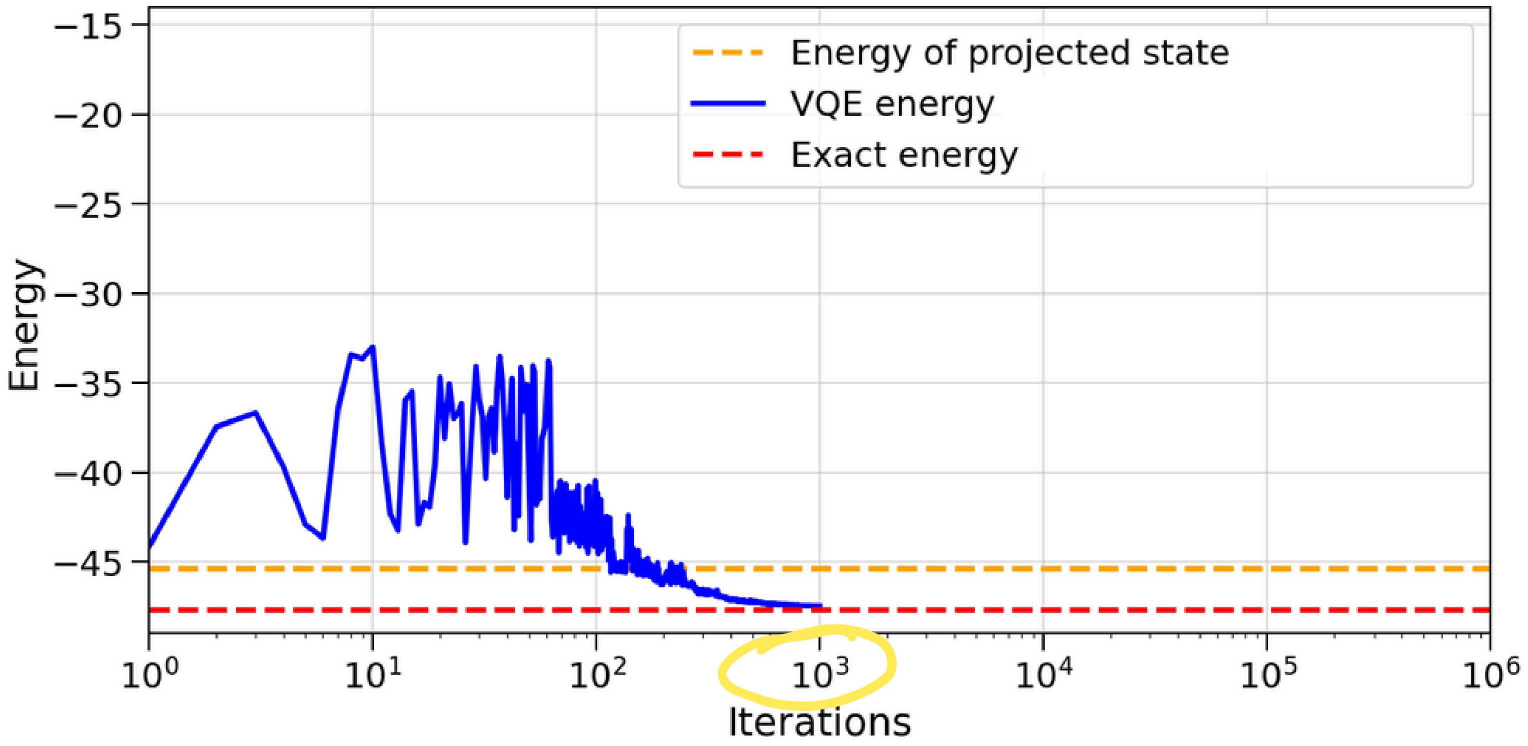


maxiter = 800,000

With Projection



CONCLUSIONS

arxiv: 2510.06702

STEP	FIDELITY
Initial product state	2.4%
J^2 Projection (11 steps)	56.7%
VQE convergence	**98.8%**

Symmetry:

- Reduces the relevant Hilbert space from 4096 to 132 states
- Improves the initial state from 2.4% to 56.7% fidelity in just 11 steps
- Guides VQE to 98.8% fidelity with the exact ground state
- Makes the algorithm 800× faster

THANK YOU FOR YOUR ATTENTION!